PRELIMINARY EXAMINATION TOPICS FOR
REAL VARIABLES

1. Basic Analysis and Topology
   Metric spaces, compactness, connectedness, continuity, series and sequences, Arzela-Ascoli theorem, Stone-Weierstrass theorem

2. Measure and Integration Theory
   Outer measure and the Caratheodory extension theorem, measures, Borel and Lebesgue measure, Egoroff’s theorem, Lusin’s theorem, integration with respect to a measure, Riemann-Stieltjes integral, Fatou’s lemma, monotone convergence theorem, dominated convergence theorem, complex and signed measure, Hahn decomposition, Jordan decomposition, Radon-Nikodym theorem, Lebesgue decomposition, Fubine-Tonelli theorem.

3. Differentiation Theory
   Functions of bounded variation, absolutely continuous functions, differentiation of monotone functions, Differentiation of Borel measures, fundamental theorem of calculus for Lebesgue integrals, Lebesgue differentiation theorem

4. Functional Analysis
   Banach spaces, Hilbert spaces, bounded linear operators and functional duals spaces, Hahn-Banach theorem, closed graph theorem, open mapping theorem, uniform boundedness principle, Baire category theorem, weak topologies

5. \textbf{L}^p Spaces
   Inequalities of Jensen, Minkowski, and Holder, completeness, dual of LP

6. Other Topics
   Basic Fourier analysis, Riesz representation theorems for various spaces of continuous functions