Probability and Neurobiology

(a really presumptuous title)

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The auditory system: from the outside in.
Latency in the auditory nerve and the lateral lemniscus.
Formulation of a simple mathematical question.
Computational and mathematical results.
Current work.
the ear
A nice mathematical problem

A real nice mathematical problem

Asin(wt) water

flexible basilar membrane

membrane compliance increases →

cochlea


the 8th nerve


The wave delay

N. Kiang, Discharge Patterns of Single Fibers in the Cat’s Auditory Nerve, MIT, 1965.
dB affects auditory nerve latency

An auditory nerve histogram
From the cochlea to the Lateral Lemniscus
frequency and dB invariance in the lateral lemniscus

A Lateral Lemniscus histogram

Covey, E., and J. Casseday, J. Neurosci., 11, 1991
once again, from the cochlea to the lateral lemniscus
once again, from the cochlea to the lateral lemniscus
once again, from the cochlea to the lateral lemniscus
compensating for the wave delay

Oertel et al, P.N.A.S., 97, 2000
stimulus

\[ n \text{ cells} \]
$f(t) = \text{probability density of firing time of input neurons}$

stimulus

$n$ cells

$\cdots$
$f(t) = \text{probability density of firing time of input neurons}$

stimulus

$n$ cells

target cell fires when it receives $m$ hits in $\varepsilon$ msec
\[ f(t) = \text{probability density of firing time of input neurons} \]

\[ g_{n,m,\varepsilon,f}(t) = \text{probability density of firing time of target neuron} \]

stimulus

\[ n \text{ cells} \]

target cell fires when it receives \( m \) hits in \( \varepsilon \) msec
experimental densities
Theorem: Let $f$ be a density and let $f \in L^{m+1}$, then

$$g_{n,m,\varepsilon,f} \xrightarrow{L^1} \frac{f^m}{\int f^m}$$

- If, in addition, we require $f$ to be left-continuous, then the convergence is also pointwise.
- If, in addition, we require $f$ to be uniformly continuous, then the convergence is also uniform.
- If, in addition, we require $f$ to be in the weighted $L^p$ space $L^{m+1,1}$, the standard deviations converge.
cubing the densities
$\varepsilon = 0$

\[ \sigma_{n,m,0} = 1, \quad \text{if } f \text{ is uniform.} \]

\[ \sigma_{n,m,0} = \frac{1}{m}, \quad \text{if } f \text{ is exponential.} \]

\[ \sigma_{n,m,0} = \frac{1}{\sqrt{m}}, \quad \text{if } f \text{ is normal.} \]

\[ \sigma_{n,m,0} = \frac{\sqrt{12}}{\sqrt{(m+2)(m+3)}}, \quad \text{if } f \text{ is hat.} \]
\[ e \to \infty \]

\[ g_{m,n,e,f}(t) \longrightarrow n! f(t) \frac{F(t)^{m-1}}{(m-1)!} \frac{(1 - F(t))^{n-m}}{(n - m)!} \]

The target cell fires when the \( m \)th hit arrives.

Order Statistics.
$\epsilon = \infty$
dependence on epsilon
dependence on m
current work

• Do neurons have time windows?
current work

- Do neurons have time windows?
- other CNS systems
current work

- Do neurons have time windows?
- other CNS systems
- error correction, information sharpening
Why is the CNS so hard?

- Single neurons?
Why is the CNS so hard?

- Single neurons?
- Large scales?
Why is the CNS so hard?

- Single neurons?
- Large scales?
- Individual differences!
Why is the CNS so hard?

- Single neurons?
- Large scales?
- Individual differences!
- Ever changing!
mathematical biology

- Purpose?
mathematical biology

- Purpose?
- How?
mathematical biology

- Purpose?
- How?
- Who? (we’re good at it!)
mathematical biology

- Purpose?
- How?
- Who? (we’re good at it!)
- Thanks!
References
