Simple Models in a Complex World

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What in the world do I do?

That’s a tough question.
What in the world do I do?

Mechanistically modeling problems arising from real world phenomena, often using the tools of dynamical systems and bifurcation theory, with an inclination for reduced ("simple") models and a slight affinity for theory.
Why reduced modeling?

Everything should be made as simple as possible, but not simpler.

Albert Einstein
Stuff I’ll be talking about today
History: Logic Gates

### AND

- **Symbol:** ∧
- **Function:** \( F(x, y) := x ∧ y \)
- **Truth Table:**
  
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( F(x, y) )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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### OR

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</table>

### NOT

- **Symbol:** \( \neg \)
- **Function:** \( F(x) := \neg x \)
- **Truth Table:**
  
<table>
<thead>
<tr>
<th>x</th>
<th>( F(x) )</th>
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<tbody>
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</tbody>
</table>

### NAND

- **Symbol:** \( \neg \cdot \)
- **Function:** \( F(x, y) := \neg (x ∧ y) \)
- **Truth Table:**
  
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( F(x, y) )</th>
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</tbody>
</table>

### NOR

- **Symbol:** \( \neg + \)
- **Function:** \( F(x, y) := \neg (x ∨ y) \)
- **Truth Table:**
  
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( F(x, y) )</th>
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History: RS flip-flop

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$S$</td>
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</table>
History: Chaotic Circuits

\[ C_1 \frac{dV_{C_1}}{dt} = G(V_{C_2} - V_{C_1}) - g(V_{C_1}) \]

\[ C_2 \frac{dV_{C_2}}{dt} = G(V_{C_1} - V_{C_2}) + \chi_L \]

\[ L \frac{d\chi_L}{dt} = -V_{C_2} \]
Chaotic NOR gate/RS flip-flop

- Some SPICE sims, but no experiments and no models (till now).
- System is too complex for traditional ODE models.

Experimentation

- Designed new version of chaotic RS flip-flop
- Built physical realization
- Simulations agree with the experiments
- Components caught on fire (oops)
- Testing the possibility of chaotic encryption
Modeling Thresholds

Experimental results:

Let $\xi$ and $\eta$ represent the threshold voltages

$$
\xi_{n+1} = f(l_1, l_2, \xi_n), \\
\eta_{n+1} = g(l_3, l_4, \eta_n);
$$

(2)

What we know from direct observation:

$$
f(0, 0, 0) = 0 \quad f(1, 0, 1) = f(0, 1, 1) = 1 \quad f(1, 1, \star) = \star, \\
g(0, 0, -1) = -1 \quad g(1, 0, 0) = g(0, 1, 0) = 0 \quad g(1, 1, \star) = \star.
$$

(3)
Assumptions

To satisfy the above observations we let

\[
\begin{align*}
  f(l_1, l_2, x_1) &:= |l_1 - l_2| + l_1 l_2 y_f(x_1), \\
  g(l_3, l_4, x_2) &:= |l_3 - l_4| - 1 + l_3 l_4 y_g(x_2);
\end{align*}
\]

- \( y \) are combinations or modifications of tent maps.
- \( y_g - 1 \) has a f.p. at \( x = 0 \).
- \( y_f \) has two nonzero f.p.'s near \( \pm 1 \).
Simulations vs. Experiments
Current and future work

- Develop a capacitor-based (rather than our TCU-based) model and compare.
- Build neuronal analogs.
- Model microfluidic circuits.
Walking in confined geometry

Figure: [Harris et al., PRE 2013]

Figure: http://math.mit.edu/~bush/?page_id=252

- (Couder et al. Nature (2005)) observed walking dynamics.
- Groups at Paris, MIT, and others have made significant experimental and theoretical contributions since.
- Analogous to wave-particle duality.
- Can produce quantum-like behavior.
Walking in rotating frame

Figure: http://math.mit.edu/~bush/?page_id=252
Simplifying assumptions:

- Droplet shifted proportionally to the gradient of the wave field.
- Droplet moving in a straight line.
- Only one eigenmode is excited in a confined geometry.

Figure: Source: [Rahman and Blackmore, CSF 2016]

\[
\begin{bmatrix}
  w_{n+1} \\
  x_{n+1}
\end{bmatrix} =
\begin{bmatrix}
  \mu[w_n + \Psi(x_n)] \\
  x_n - Cw_n \Psi'(x_n)
\end{bmatrix}
\] (5)
Simulations

Figure: $\mu = 0.94$
Confining the walker

\[ w(n + 1) = \mu[w_n + \Psi(x_n)] \]
\[ x(n + 1) = [x_n - Cw_n\Psi'(x_n)] \mod 2\pi \]

\[ \Psi(x, \beta) = \frac{\cos \beta}{\sqrt{\pi}} \sin 3(x - pi/2) + \frac{\sin \beta}{\sqrt{\pi}} \sin 5(x - pi/2) \]

Figure: Source: [Filoux, Hubert, Vanderwalle. Phys. Rev. E(2015)]
Current and future work

- Working on kicked rotator - like model on the annulus.
- Working on model for elliptical corral.
- Connections between dynamics and statistics.
Drug diffusion: Story time

Two Bengalis, a grant, and Reddit.

- Souparno’s NIH grant on individualized treatment.
- Lots of procrastination.
- Redditors write bad science headlines.
Drug diffusion: Story time
Two Bengalis, a grant, and Reddit.

- This title is much better.
- Using alcohol to treat solid tumors.

Development of enhanced ethanol ablation as an alternative to surgery in treatment of superficial solid tumors

Robert Morhard, Corrine Nief, Carlos Barrero Castedo, Fangyao Hu, Megan Madonna, Jenna L. Mueller, Mark W. Dewhirst, David F. Katz & Nirmala Ramanujam

*Scientific Reports* 7, Article number: 8750 (2017)  
doi:10.1038/s41598-017-09371-2

Received: 05 May 2017  
Accepted: 27 July 2017  
Published online: 18 August 2017
Simplest possible mechanistic model

Assumptions:

- Spherical solid tumor.
- Diffusion with constant diffusivity.
- Leaky boundary.
- Injection into the center.
- Diffusion begins after injection ends.
- There exists a minimum drug concentration required to kill a cell.
Simplest possible mechanistic model

\[
\frac{\partial u}{\partial t} = D \left( \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} \right) u(r, t = 0)
\]

\[
D \frac{\partial u}{\partial r} \bigg|_{r=R} = -\gamma u(r = R, t)
\]
Cell death

Radial Diffusion

![Graph showing concentration profile with radius on the x-axis and concentration on the y-axis.]

Concentration Profile

Radius

Concentration
Diffusion
No ablation
Diffusion

Partial ablation
Diffusion
Full ablation
Current and future work

- Model the relationship between time and threshold.
- Using statistics to connect the model with population data.
- Making individualized predictions.
- Analyze our other models.
What came first, the experiment or the model?

Solitary Waves

- First discovered by John Scott Russell.
- Not just water waves;
  - Generally solitary waves are solutions to non-linear evolutionary PDEs that translate at a constant speed while maintaining its profile.
Kruskal-Zabusky (1960): Solitary waves of KdV
\(u_t + 6uu_x + u_{xxx} = 0\) preserve shape and velocity after collisions.

Soon after similar studies on NLS \(iu_t + u_{xx} + |u|^2u = 0\) and Sine-Gordon \(u_{tt} - u_{xx} + \sin u = 0\).

“Easy” because they are completely integrable.
In the 70s and 80s many studies on $\varphi^4$ ($\varphi_{tt} - \varphi_{xx} + \varphi - \varphi^3 = 0$) were conducted.

- Much more difficult because it’s not integrable.
- Many numerical experiments on kink-antikink (colliding in a special way) solutions (some observations dubbed “chaotic scattering”).
Recent History

- Reduction of PDE ($\varphi^4$) to a system of ODEs that approximates the behavior.

\[
\begin{align*}
    m\ddot{X}(t) + U'(X) + cAF'(X) &= 0, \\
    \ddot{A}(t) + \omega^2 A + cF(X) &= 0; \\
    U(X) &= e^{-2X} - e^{-X}, \\
    F(X) &= e^{-X}
\end{align*}
\]  

(6)

where $X$ is the position of the center of a wave and $A$ is the amplitude.

Figure: Source: [Goodman and Haberman. *Phys. Rev. Lett.* (2007)]
Experiment


Experimental set up:

Movies:
If anyone is interested in knowing more about my work please visit my website: myweb.ttu.edu/aminrahm

Aminur Rahman

Welcome to my website.
A summary of my academic and extracurricular activities. If you don’t find what you are looking for feel free to contact me. Also, you may call me Amin for short.

Research

I am interested in modeling and analyzing various real world phenomena. Some of the topics I have worked on and/or have an interest in working on are: Hydrodynamic Quantum Analogs, Logical circuit dynamics, Chaotic scattering, Cancer modeling, and Particle Accelerator Physics. To find out more about my current research please refer to my research statement.

Cancer Drug Response:

And, thank you for your attention!