Investing with Momentum: The Past, Present, and Future

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Expected returns on assets are not completely explained by using only historical means (and standard deviations) in the United States and Japan. The reported financial data stock selection model Guerard and Takano [1991] presented in this journal was complemented with earnings expectations data in Guerard [2006]. In this analysis, we include a price momentum variable in a regression-based stock selection model. We construct and estimate a U.S. stock selection model using earnings expectations data, price momentum, and reported financial data for the period from January 1985 to December 2007. Despite the recent volatility of the momentum factor, momentum is still statistically associated with security returns and can be used with other factors to rank-order stocks for purchase. A composite value of momentum, value, and growth factors is estimated for U.S. equities universe to identify potentially mispriced stocks. In addition the regression-weighting of factors enhanced information coefficients relative to equal-weighted factors. Thus, momentum and analysts' forecast variables dominate the regression-based composite model of expected returns. We created portfolios for the January 1998–December 2007 period. We report three conclusions: 1) momentum investing has been rewarded by the market in the United States from December 1928 to December 2007; 2) momentum can be combined with fundamental data, such as earnings, book value, cash flow, and sales, and earnings forecast revisions to identify undervalued securities; 3) alternative multifactor models are useful in producing portfolios that offer potential outperformance of a broad U.S. equity index, the Russell 3000 Growth Index. This study addresses several issues:

1. how momentum has been analyzed and tested by academicians and practitioners, who examined how momentum fits with the academic literature on expected returns modeling;
2. how momentum can be integrated into a stock selection model;
3. the construction of efficient portfolios using the composite stock selection model;
4. testing the efficient portfolio returns to examine whether the returns are statistically different from an average model portfolio return;
EXPECTED RETURNS MODELING AND STOCK SELECTION MODELS: THE ADDITION OF PRICE MOMENTUM

This analysis builds upon Guerard and Takano [1991] and Guerard [2006]. We refer the reader to those studies for much of the underlying expected returns literature. There are many approaches to security valuation and the creation of expected returns. The first approaches to security analysis and stock selection involved the use of valuation techniques using reported earnings and other financial data. Graham and Dodd [1934] recommended that stocks be purchased on the basis of the price/earnings ratio (P/E) and Basu [1977] reported evidence supporting the low P/E model. Hawawini and Keim [1995] found statistical support for the high E/P variable of NYSE and AMEX stocks from April 1962–December 1989. Guerard and Takano used book value, cash flow, and sales, relative to price, in their analysis. The major papers on combination of value ratios to predict stock returns that include at least CP and/or SP include Chan, Hamao, and Lakonishok [1991]; Bloch, Guerard, Markowitz, Todd, and Xu [1993]; Lakonishok, Shleifer, and Vishny [1994]; and Haugen and Baker [2010]. In fact, the Bloch et al. [1993] paper was a more technical version of Guerard and Takano [1991].

Earnings forecasting enhances returns relative to using only reported financial data and valuation ratios. In 1975, a database of earnings per share (EPS) forecasts was created by Lynch, Jones, and Ryan, a New York brokerage firm, by collecting and publishing the consensus statistics of one-year-ahead and two-year-ahead EPS forecasts (Brown [1999]). The database evolved to become the Institutional Brokerage Estimation Service (I/B/E/S) database.

There is an extensive literature regarding the effectiveness of analysts’ earnings forecasts, earnings revisions, earnings forecast variability, and breadth of earnings forecast revisions, summarized in Bruce and Epstein [1994], Brown [1999], and Ramnath, Rock, and Shane [2008]. The vast majority of the earnings forecasting literature in the Bruce and Brown references find that the use of earnings forecasts do not increase stockholder wealth, as specifically tested in Elton, Gruber, and Gultekin [1981] in their consensus forecasted growth variable, FGR. Reported earnings follow a random walk with drift process, and analysts are rarely more accurate than a no-change model in forecasting earnings per share (Cragg and Malkiel [1968]). Analysts become more accurate as time passes during the year and quarterly data are reported. Analyst revisions are statistically correlated with stockholder returns during the year (Hawkins, Chamberlain, and Daniel [1984]; Arnott [1985]). Wheeler [1994] developed and tested a strategy in which analyst forecast revision breadth, defined as the number of upward forecast revisions less the number of downward forecast revisions divided by the total number of estimates, was the criteria for stock selection. Wheeler found statistically significant excess returns from the breadth strategy. A composite earnings variable, CTEF, is calculated using equal-weighted revisions, RV, forecasted earnings yields, FEP, and breadth, BR, of FY1 and FY2 forecasts, a variable put forth in Guerard [1997] and further tested in Guerard, Gultekin, and Stone [1997].

Adding I/B/E/S variables to the eight value ratios in Guerard and Takano [1993] produced more than 2.5% of additional annualized return (Guerard, Gultekin, and Stone [1997]). The finding of significant predictive performance value for I/B/E/S variables indicates that analyst forecast information has value beyond purely statistical extrapolation of past value and growth measures. Possible reasons for the additional performance benefit could be that analysts’ forecasts and forecast revisions reflect information in other return-pertinent variables, or discontinuities from past data, or serve as a quality screen on otherwise out-of-favor stocks. The quality screen idea would confirm Graham and Dodd’s argument that value ratios should be used in the context of the many qualitative and quantitative factors that they argue are essential to informed investing. To test the risk-corrected performance value of the forecasts, Guerard, Gultekin, and Stone [1997] formed quarterly portfolios with risk being modeled via a four-factor APT-based model (created using five years of past monthly data). The portfolios’ quarterly returns averaged 6.18% before correcting for risks and transaction costs with excess returns of 3.6% after correcting for risk and 2.6% quarterly after subtracting 100 bps to reflect an estimate of two-way transactions costs. Guerard [2006] reported the growing importance of earnings forecasts, revisions, and breadth in Japan and the U.S., particularly with respect to smaller-capitalized securities.

Momentum investing was studied by academics at about the same time that earnings forecasting studies were being published. Arnott [1979] and Brush and Boles
[1983] found statistically significant power in relative strength. The Brush and Boles analysis was particularly valuable because it found that the short-term (three-month) financial predictability of a naïve monthly price momentum model, taking the price at time \( t - 1 \) divided by the price 12 months ago, \( t - 12 \), was as statistically significant at identifying underpriced securities as using the alpha of the market model adjusted for the security beta. Brush and Boles found that beta adjustments slightly enhanced the predictive power in the 6- to 12-month periods. Brush [2001] is an excellent 20-year summary of the price momentum literature. Fama and French [1992, 1995] employed a price momentum variable using the price 2 months ago divided by the price 12 months ago, thus avoiding the well-known return or residual reversal effect. The Brush, Korajczyk, and Sadka [2004] and Fama studies find significant stock price anomalies, even with Korajczyk and Sadka using transactions costs. The vast majority find that the use of 3-, 6-, and 12-month price momentum variables, often defined as intermediate-term variables, are statistically significantly associated with excess returns. Brush [2001] reports that the quarterly information coefficient (IC) of the three-month price momentum variable exceeds the monthly IC, 0.073 versus 0.053.

Moreover, if one truly wants to test the price momentum concept, one need only use the Center for Research in Security Prices (CRSP) database and test a variation of the Brush and Boles [1983] price momentum (PM) variable back to December 1927. One need only examine the monthly ICs of the CRSP PM variable for the December 1927–December 2007 period, as shown in Exhibit 1. The overall information coefficient is 0.040 for the 79-year period and is highly statistically significant. However, there is considerable variation, as shown in Exhibit 2, where we report the monthly estimated \( t \)-statistics of the ICs.

Momentum is well associated with excess returns in the academic and practitioner-oriented literature, particularly in the works of Jack Brush. Guerard and Takano [1991] and Bloch, Guerard, Markowitz, Todd, and Xu [1993] used an eight-factor model for stock selection. If we add a Brush-based price momentum, taking the price at time \( t - 1 \) divided by the price 12 months ago, \( t - 12 \), denoted PM, and the consensus analysts' earnings forecasts and analysts' revisions composite variable, CTEF, to the stock selection model, we can estimate an expanded stock selection model to use as an input to an optimization analysis. The stock selection model estimated in this study, denoted as United States Expected Returns, \( \text{USER} \), is:

\[
TR_{t+1} = a_0 + a_1 EP_t + a_2 BP_t + a_3 CP_t + a_4 SP_t + a_5 RBP_t + a_6 RCP_t + a_7 RSP_t + a_8 CTEF_t + a_9 PM_t + \epsilon_t
\]  

(1)
EXHIBIT 2

Source: Ratios calculated using CRSP data.

Where

\[
EP = \frac{\text{earnings per share}}{\text{price per share}} = \text{earnings-price ratio;}
\]

\[
BP = \frac{\text{book value per share}}{\text{price per share}} = \text{book-price ratio;}
\]

\[
CP = \frac{\text{cash flow per share}}{\text{price per share}} = \text{cash flow-price ratio;}
\]

\[
SP = \frac{\text{net sales per share}}{\text{price per share}} = \text{sales-price ratio;}
\]

\[
REP = \frac{\text{current EP ratio}}{\text{average EP ratio over the past five years;}}
\]

\[
RBP = \frac{\text{current BP ratio}}{\text{average BP ratio over the past five years;}}
\]

\[
RCP = \frac{\text{current CP ratio}}{\text{average CP ratio over the past five years;}}
\]

\[
RSP = \frac{\text{current SP ratio}}{\text{average SP ratio over the past five years;}}
\]

\[
CTEF = \text{consensus EPS I/B/E/S forecast, revisions, and breadth;}
\]

\[
PM = \text{price momentum; and}
\]

\[
e = \text{randomly distributed error term.}
\]

The USER model is estimated using weighted latent root regression, WLRR, analysis on Equation (1) to identify variables statistically significant at the 10% level; uses the normalized coefficients as weights; and averages the variable weights over the past 12 months. The 12-month smoothing is consistent with the four-quarter smoothing in Guerard and Takano [1991] and Bloch et al. [1993].

While the EP and BP variables are significant in explaining returns, the majority of the forecast performance is attributable to other model variables, namely the relative earnings-to-price, the relative cash-to-price, relative sales-to-price, the price momentum variable, and the earnings forecast variable. The consensus earnings forecasting variable, CTEF, and the price momentum variable, PM, dominate the composite model, as is suggested by the fact that the variables account for 45% of the model average weights.\(^1\)

In terms of information coefficients, ICs, the use of the WLRR procedure produces the higher IC for the models during the 1985–2007 time period, 0.047, versus the equal-weighted IC of 0.040, a result consistent with the previously noted studies. The IC test of statistical significance can be referred to as a Level I test.

We have briefly surveyed the academic literature on anomalies and found substantial evidence that valuation, earnings expectations, and price momentum variables are significantly associated with security returns. Further evidence on the anomalies is found in Levy [1999].\(^2\)
EFFICIENT PORTFOLIO CONSTRUCTION

The USER model can be input into the MSCI Barra system to create optimized portfolios. The equity factor returns \( f_k \) in the Barra United States Equity Risk Model, denoted USE3, are estimated by regressing the local excess returns, \( r_n \), against the factor exposures, \( X_{nk} \),

\[
r_n = \sum_{k=1}^{K} X_{nk} f_k + u_n
\]  

(2)

USE3 uses weighted least squares, assuming that the variance of specific returns is inversely proportional to the square root of total market capitalization. The USER model is our approximation of the expected return, or the forecast active return, \( \alpha \), of the portfolio. Researchers in industry most often apply the Markowitz mean–variance framework to active management, as described in Grinold and Kahn [1999]:

\[
U = \alpha \cdot h - \lambda \cdot \omega^2 \cdot h^2
\]  

(3)

Here \( \alpha \) is the forecast active return (relative to a benchmark which can be cash), \( \omega \) is the active risk, and \( h \) is the active holding (the holding relative to the benchmark holding). By varying the tolerance or risk aversion, one can create the efficient frontier in the Barra model, as was done in Bloch et al. [1993], by varying the variable \( m \). The risk aversion parameter, \( \lambda \), captures individual investor preference. Grinold and Kahn [1999] use the information ratio (IR) as a portfolio construction objective to be maximized, which measures the ratio of residual return to residual risk:

\[
IR = \frac{\alpha}{\omega}
\]  

(4)

We construct an efficient frontier varying the risk-aversion levels. The portfolio construction process uses 8% monthly turnover, after the initial portfolio is created, and 125 bps of transactions costs each way. The USER-optimized portfolios outperform the market, here defined as the Russell 300 Growth Index, denoted R3G. The portfolio that maximizes the geometric mean (Markowitz [1976]) occurs at a risk-aversion level of 0.02. The Sharpe ratio also is maximized at a risk-aversion level of 0.02 with 89 stocks in the efficient portfolio. The regression-weighted model, USER, outperforms its equal-weighted corresponding model, EQ, in terms of the Sharpe ratio, information ratio, geometric mean, and the \( t \)-value on asset selection. The dominance of regression-weighting was reported in Bloch et al. [1993]. The information ratio, defined as the ratio of portfolio excess return relative to estimated tracking error, is maximized at a risk-aversion level of 0.01; see Exhibit 3.

The efficient USER portfolio at a risk-aversion level of 0.02 offers exposure to MSCI Barra-estimated

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**EXHIBIT 3**


<table>
<thead>
<tr>
<th>RAL</th>
<th>Model</th>
<th>TManaged</th>
<th>STD</th>
<th>TActive</th>
<th>t-Active</th>
<th>AssetSel</th>
<th>t-AS</th>
<th>t-Rskl</th>
<th>t-Rskl</th>
<th>IR</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>EQ</td>
<td>12.480</td>
<td>19.962</td>
<td>8.663</td>
<td>2.475</td>
<td>7.445</td>
<td>3.164</td>
<td>4.468</td>
<td>1.545</td>
<td>0.783</td>
<td>-1.176</td>
</tr>
<tr>
<td>0.01</td>
<td>USER</td>
<td>13.263</td>
<td>20.844</td>
<td>9.446</td>
<td>2.720</td>
<td>8.265</td>
<td>3.300</td>
<td>3.555</td>
<td>1.373</td>
<td>0.860</td>
<td>-1.220</td>
</tr>
<tr>
<td>0.02</td>
<td>EQ</td>
<td>10.993</td>
<td>20.193</td>
<td>7.176</td>
<td>2.267</td>
<td>6.526</td>
<td>2.980</td>
<td>3.759</td>
<td>1.458</td>
<td>0.717</td>
<td>-1.042</td>
</tr>
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<td>0.02</td>
<td>USER</td>
<td>12.517</td>
<td>20.494</td>
<td>8.700</td>
<td>2.779</td>
<td>7.490</td>
<td>3.281</td>
<td>3.214</td>
<td>1.394</td>
<td>0.879</td>
<td>-1.050</td>
</tr>
<tr>
<td>0.03</td>
<td>EQ</td>
<td>10.612</td>
<td>20.155</td>
<td>6.796</td>
<td>2.230</td>
<td>6.270</td>
<td>2.945</td>
<td>3.564</td>
<td>1.440</td>
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<td>-0.990</td>
</tr>
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<td>0.03</td>
<td>USER</td>
<td>11.651</td>
<td>20.578</td>
<td>7.835</td>
<td>2.660</td>
<td>6.956</td>
<td>3.220</td>
<td>2.901</td>
<td>1.330</td>
<td>0.839</td>
<td>-0.980</td>
</tr>
<tr>
<td>0.05</td>
<td>EQ</td>
<td>9.155</td>
<td>19.992</td>
<td>5.339</td>
<td>2.054</td>
<td>5.300</td>
<td>2.848</td>
<td>2.640</td>
<td>1.280</td>
<td>0.650</td>
<td>-0.787</td>
</tr>
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<td>0.05</td>
<td>USER</td>
<td>9.355</td>
<td>20.346</td>
<td>5.539</td>
<td>2.360</td>
<td>5.742</td>
<td>3.226</td>
<td>2.014</td>
<td>1.234</td>
<td>0.745</td>
<td>-0.764</td>
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<td>0.09</td>
<td>EQ</td>
<td>7.839</td>
<td>19.982</td>
<td>4.022</td>
<td>1.878</td>
<td>4.101</td>
<td>2.578</td>
<td>2.115</td>
<td>1.282</td>
<td>0.594</td>
<td>-0.582</td>
</tr>
<tr>
<td>0.09</td>
<td>USER</td>
<td>8.170</td>
<td>20.161</td>
<td>4.353</td>
<td>2.194</td>
<td>5.214</td>
<td>3.342</td>
<td>1.610</td>
<td>1.147</td>
<td>0.694</td>
<td>-0.598</td>
</tr>
<tr>
<td>0.15</td>
<td>EQ</td>
<td>6.540</td>
<td>19.979</td>
<td>2.723</td>
<td>1.575</td>
<td>3.265</td>
<td>2.408</td>
<td>1.629</td>
<td>1.233</td>
<td>0.498</td>
<td>-0.421</td>
</tr>
<tr>
<td>0.15</td>
<td>USER</td>
<td>7.122</td>
<td>19.800</td>
<td>3.305</td>
<td>1.967</td>
<td>4.358</td>
<td>3.197</td>
<td>1.349</td>
<td>1.169</td>
<td>0.622</td>
<td>-0.477</td>
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<tr>
<td>0.20</td>
<td>EQ</td>
<td>5.814</td>
<td>19.848</td>
<td>1.997</td>
<td>1.322</td>
<td>2.822</td>
<td>2.294</td>
<td>1.369</td>
<td>1.184</td>
<td>0.418</td>
<td>-0.356</td>
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<tr>
<td>0.20</td>
<td>USER</td>
<td>6.617</td>
<td>19.641</td>
<td>2.801</td>
<td>1.824</td>
<td>4.007</td>
<td>3.174</td>
<td>1.191</td>
<td>1.143</td>
<td>0.577</td>
<td>-0.416</td>
</tr>
</tbody>
</table>

Notes: RAL = risk aversion level; TManaged = total managed return; TActive = total active return; AssetSel = asset selection; IR = information ratio. Size, MOM, EY, Value, Growth are BARRA multifactor risk exposures. EQ = EQ-WT(EP, BP, SP, REP, RBP, RCP, RSP, PM, CTEF); USER = WLRR-WT(EP, BP, CP, SP, REP, RBP, RCP, RSP, PM, CTEF).
momentum, value, and growth exposures; see Exhibit 4. The reader is hardly surprised with these exposures, given the academic literature and stock selection criteria and portfolio construction methodology employed.

A complete MSCI Barra portfolio performance attribution is shown in Exhibit 5. Let us examine the portfolio corresponding to a risk-aversion level portfolio of 0.02.

The authors obtained similar results when an S&P 500 benchmark was used and the results are available from the authors. The Barra model is one of the most-used institutional asset management tools, and alternative risk models can be used in portfolio construction. In Exhibit 6, we report alternative risk models such as the Blin and Bender [1995] APT risk model using a lambda of 200 (the inverse of the risk-aversion measure) using mean–variance (MV) and tracking error at risk (TaR) models and the Cognity model that maximizes the mean-expected tail loss ratio (M-ETL), as developed in Rachev et al. [2010]. The estimated APT and Cognity t-statistics on asset selection are comparable to the Barra-derived t-statistic on asset selection.

The creation of portfolios with a multifactor model and the generation of excess returns will hereby be referred to as a Level II test of portfolio construction.

**A FURTHER TEST OF DATA MINING CORRECTIONS**

In the (practical) world of Wall Street, it is conventional wisdom to cut your historical backtested excess returns in half; that is, if your backtested excess return (the portfolio geometric mean return less the geometric mean of the benchmark) was 6%, or 600 bps, an investment manager and/or a client might expect 3% excess returns in the future. In January 1991, Harry
Markowitz and his Daiwa Global Portfolio Research Department launched Fund Academy, a Japanese-only, Tokyo-based investment strategy. In its first year, ending December 1991, the Daiwa Portfolio Optimization System (DPOS) outperformed the benchmark by some 700 bps. Markowitz asked “Did we get lucky?” Ganlin Xu, the Daiwa Global Research Department mathematician, developed a testing methodology that Markowitz and Xu [1994] published in *The Journal of Portfolio Management*. A secondary question is, Could a manager have obtained a similar portfolio return using other models, rather than the model actually employed? Let us trace the development of the Markowitz and Xu model and estimate the data mining corrections estimator for a series of U.S. expected returns models.

Let $G_{M}$ be the backtested geometric best of the “best” historical simulation during $T$ periods. Markowitz and Xu [1994] work with the logarithm of the geometric mean.

$$ g_{b} = \log (1 + G_{M}) \quad (5) $$

The Markowitz and Xu data mining corrections (DMC) test assumes that the $T$ period historical returns were identically and independently distributed (i.i.d.), and that future returns are drawn from the same population (also i.i.d.). Because we test many models, not just the best model, the geometric mean is no longer the best unbiased estimate of the true, underlying population $g_{b}$.

Markowitz and Xu [1994] set $y_{i}$ as the logarithm of one plus the return for the $i^{th}$ portfolio selection model in period $t$. $y_{i}$ is

$$ y_{i} = \mu_{i} + z_{i} + \varepsilon_{i} \quad (6) $$

where $\mu_{i}$ is a model effect, $z_{i}$ is a period effect, and $\varepsilon_{i}$ is a random deviation.

Markowitz and Xu [1994] assume that the period return is observable, as is the return of a market index. In this case, $r_{i}$ is an excess return of model $i$:

$$ r_{i} = y_{i} - z_{i} = \mu_{i} + \varepsilon_{i} \quad (7) $$

The random deviation of the return, $\varepsilon_{i}$, has a zero mean and is uncorrelated with the model effect and $\mu_{i}$ and other model random effects. In the initial Markowitz and Xu model, excess returns are assumed to be independent of time and the error terms of other variables (models) used in portfolio construction.

Markowitz and Xu [1994] estimated a model (Model III), in which $y_{i} = \mu_{i} + \varepsilon_{i}$, as in Equation (7). Markowitz and Xu did not require that models be independent of one another. Thus, the covariance of $(\varepsilon_{i}, \varepsilon_{j})$ need not be zero. Thus, the Markowitz–Xu Model III is not only the general case (Model I being a special case of Model III), but Model III is consistent with testing in business-world portfolio construction and testing. Finally, the appropriate estimate of $\mu_{i}$ in Model I is not the average return

$$ \bar{r} = \frac{1}{T} \sum_{t=1}^{T} r_{i} \quad (8) $$

but rather a combination with the average of average returns

$$ \bar{r} = \frac{1}{T} \sum_{t=1}^{T} \bar{r}_{i} \quad (9) $$

The estimate of $\mu_{i}$ is regressed back to the average return (the grand average),

$$ \hat{\mu} = \bar{r} + \beta (\bar{\mu} - \bar{r}) \quad (10) $$

where $0 < \beta < 1$.

The best linear estimate of the unknown $\mu_{i}$ is

$$ \hat{\mu}_{i} = E\mu + \beta (\bar{r} - E\mu) \quad (11) $$

$$ \beta = \frac{\text{cov}(\bar{r}, \mu)}{\text{Var}(\bar{r})} \quad (12) $$

### Exhibit 6

**USER Analysis, January 1998–December 2007**

<table>
<thead>
<tr>
<th>USER Portfolio Construction</th>
<th>TManaged</th>
<th>STD</th>
<th>Asset Selection</th>
<th>t(AssetSel)</th>
<th>IR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognity_M-ETL</td>
<td>15.65</td>
<td>14.05</td>
<td>8.60</td>
<td>3.69</td>
<td>1.26</td>
</tr>
<tr>
<td>APT, L = 200, TaR</td>
<td>14.59</td>
<td>18.08</td>
<td>11.38</td>
<td>3.78</td>
<td>1.23</td>
</tr>
<tr>
<td>APT L = 200, MV</td>
<td>13.51</td>
<td>17.37</td>
<td>9.72</td>
<td>3.66</td>
<td>1.21</td>
</tr>
</tbody>
</table>

*Notes: Monthly turnover = 8%; 125 bps of transactions costs each way.*
Thus, $\beta$ is the regression coefficient of \( m \) as a function of \( r_i \).

The Markowitz–Xu DMC test did not use a “holdout period,” as they can be routinely data mined as well. That is, one can vary the estimation and holdout periods to generate the desired conclusion. Markowitz and Xu [1994] tested the DPOS strategies in Bloch et al. [1993], and the best model is illustrated in Exhibit 7. Markowitz and Xu reported a Model III $\beta$ of 0.59, which was statistically significant; that is, approximately 59% of the excess returns could be expected to continue. Alternative portfolio models were created using the factors used in the original Bloch et al. [1993] and Markowitz–Xu [1994] and I/B/E/S factors (FGR, BR, RV, FEP, CTEF) and the dividend yield, DP. The USER variable analysis passes the data mining corrections test criteria for the U.S. market, indicating that the stock selection and portfolio construction methodologies produce superior returns that are not due to chance. The USER variable, when compared to the average of most models shown in Exhibit 7, has a data mining corrections coefficient of 0.74 and is highly statistically significant, having a F-value of 3.791. Thus, one could expect 74% of the excess returns of the USER model relative to the average return to be continued. More importantly, the USER model produced a higher geometric mean than did an average model geometric mean that could have been used to manage an equity portfolio in the U.S. equity market during the January 1998–December 2007 period. The Barra-estimated models use a risk-aversion level of 0.02, 8% monthly turnover, and 125 bps of transactions costs (each way). The application of the Markowitz–Xu [1994] data mining corrections test will be referred to as a Level III test.

One sees from Exhibit 7 that the geometric mean and Sharpe ratio are higher with cash flow to price (CP), sales to price, SP, and USER variables. Markowitz [1976] and Bloch et al. [1993] advocated maximizing the geometric mean and Sharpe ratio to maximize terminal wealth. The USER model passes all three levels of hypothesis testing: 1) the model and its components have statistically significant information coefficients; 2) the strategy produces excess returns after transactions costs; and 3) the strategy produces a (significantly) higher geometric mean than the average model that could have been used to manage assets in the universe. Backtesting can never be perfect but it can be statistically significant.

### Exhibit 7


<table>
<thead>
<tr>
<th>Model</th>
<th>Geometric Mean (GM)</th>
<th>Standard Deviation (STD)</th>
<th>Sharpe Ratio (SR)</th>
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<td>R3G</td>
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An investment manager estimates an IC over the 1998–2007 period and would be reasonably confident that over the long run, a USER strategy could produce high relative returns and satisfy its clients. However, the USER model has a large loading on the Barra momentum factor in the United States Equity Risk Model (USE3). This is perfectly understandable since the USER has a large momentum variable weight. The period from June 2008 through May 2009 has been volatile to models loading on the Bara USE3 momentum factor, particularly in March–May 2009. The –0.0812 success factor return in April 2009 is shown in Exhibit 8. The April 2009 return was one of the two worst months in the 35 years of the Barra United States equity risk models. In fact, it was more than a 5.0 standard deviation movement. Markowitz discusses investment in the long-run in Chapter 5 of *Portfolio Selection* and describes a somewhat random (roulette wheel) generating process of monthly returns. The January 2009–May 2009 period of BARRA success factor returns do not appear to be random events. One can probably expect mean-reversion of the Barra...
United States Equity Model, USE3, momentum factor returns, which would be consistent with Exhibit 8.

The annualized 12-month moving average Barra-estimated Momentum factor returns are shown in Exhibit 9. The annualized moving average returns illustrate the ongoing challenges of a U.S.-based momentum strategy.

One builds and estimates stock selection models such that long-run portfolio geometric means are maximized (Markowitz [1959, 1976]). A U.S. equity strategy that has exposure to the Barra momentum factor during the January 2009–May 2009 experienced one of the worst performance periods of factor exposure in the 35 years of the model estimation. Factor mean-reversion is expected. Initial statistical results indicate that despite the statistical identification of outliers, including the pronounced level shift of April 2009, a random walk with drift adequately describes the MSCI Barra momentum returns. The timing on such a reversion is the subject of on-going research.

CONCLUSIONS

Investing with fundamental, expectations, and momentum variables is a good investment strategy over the long-run. Stock selection models often use momentum, analysts’ expectations, and fundamental data. We find support for composite modeling using these sources of data. We find additional evidence to support the use of MSCI Barra and other multifactor models for portfolio construction and risk control. We develop and estimate three levels of testing for stock selection and portfolio construction. The uses of multifactor risk-controlled portfolio returns allow us to reject the data mining corrections test null hypothesis. The anomalies literature can be applied in real-world portfolio construction.
The author acknowledges the suggestions and comments of Harry Markowitz. The author worked with Dewitt Miller of MSCI Barra to create portfolios in exhibit 3.

1 Asness, Moskowitz, and Pedersen [2008] finds that an equal-weighted composite model of 50/50 value (as measured by book value) and momentum is effective in modeling equity returns. Professor Larry Fisher, of Rutgers University, and the co-developer of the CRSP database, told Guérard during his 1991 CRSP seminar presentation that the value-based strategy of Equation (1) was merely a test of the CRSP momentum strategy. The estimation of Equation (1) addresses more completely the answer to Professor Fisher’s observation.

2 Haugen and Baker [2010] extended their 1996 study in a recent volume to honor Harry Markowitz. Haugen and Baker estimate their model using weighted least squares. In a given month they estimated the payoffs to a variety of firm and stock characteristics using a weighted least squares multiple regression in each month in the period 1963 through 2007. In the manner of Fama and MacBeth [1973], they then compute the average values for the monthly regression coefficients (payoffs) across the entire period. Dividing the mean payoffs by their standard errors we obtain t-statistics. The values for the most significant factors are computed as follows:

- Residual Return is last month’s residual stock return unexplained by the market.
- Cash Flow to Price is the 12-month trailing cash flow per share divided by the current price.
- Earnings to Price is the 12-month trailing EPS divided by the current price.
- Return on Assets is the 12-month trailing total income divided by the most recently reported total assets.
- Residual Risk is the trailing variance of residual stock return unexplained by market return.
- 12-Month Return is the total return for the stock over the trailing 12 months.
- Return on Equity is the 12-month trailing earnings per share divided by the most recently reported book equity.
- Volatility is the 24-month trailing volatility of total stock return.
• Book to Price is the most recently reported book value of equity divided by the current market price.
• Profit Margin is 12-month trailing earnings before interest divided by 12-month trailing sales.
• 3-Month Return is the total return for the stock over the trailing 3 months.
• Sales to Price is 12-month trailing sales-per-share divided by the market price.

Last month’s residual return and the return over the preceding three months have negative predictive power relative to next month’s total return. This may be induced by the fact that the market tends to overreact to most information. The four measures of cheapness: cash to price, earnings to price, book to price, and sales to price, all have significant positive payoffs. Haugen and Baker find statistically significant results for the four fundamental factors as did the previous studies we reviewed. Haugen and Baker present optimization analysis to support their stock selection modeling, and portfolio trading is controlled through a penalty function. When available, the optimizations are based on the largest 1,000 stocks in the database. Estimates of portfolio volatility are based on the full covariance matrix of returns to the 1,000 stocks in the previous 24 months. Trading costs were not reflected in the Haugen and Baker optimization analysis; however, the Haugen and Baker portfolios outperformed the benchmark by almost 5% with average annual turnover of 80% during the 1965–2007 period. Haugen and Baker noted that the t-scores are large as compared to those obtained by Fama and MacBeth even though the length of the time periods covered by the studies is comparable. The Haugen and Baker [2010] analysis and results are consistent with the Bloch et. al. [1993] model.

Guerard, Chettiappan, and Xu [2010] rejected the null hypothesis of data mining using the Markowitz–Xu test for a 22-country non-U.S. universe (EAFE plus Canada) for the October 1995–May 2008 period using the MQ variable, an equal-weighted composite strategy of price momentum and analysts’ revisions.

REFERENCES


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