Sample Final

1-4: the mean $\bar{x} = \frac{\sum x_i}{n} = \frac{10+17+(-2)+10+23+26}{6} = 14$ (1B)

x_i	$(x_i - \bar{x})^2$
10	$(10-14)^2$
17	$(17-14)^2$
-2	$(-2-14)^2$
10	$(10-14)^2$
23	$(23-14)^2$
26	$(26-14)^2$
	522

The std. dev. $S = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{522}{5}} = 10.22$ (2D)

ordered obs. $-2, 10, 10, 17, 23, 26$
$\quad r_1\ r_2\ r_3\ r_4\ r_5\ r_6$

For Q_1: $LQ_1 = \frac{25}{100} * 7 = 1.75 \Rightarrow Q_1 = r_1 + 0.75(r_2 - r_1) = -2 + 0.75 * 12 = 7$

For Q_2: $LQ_2 = \frac{50}{100} * 7 = 3.5 \Rightarrow Q_2 = r_3 + 0.5(r_4 - r_3) = 10 + 0.5 * 7 = 13.5$

For Q_3: $LQ_3 = \frac{75}{100} * 7 = 5.25 \Rightarrow Q_3 = r_5 + 0.25(r_6 - r_5) = 23 + 0.25*3 = 23.75$ (3.A)

5# summary: $-2, 7, 13.5, 23.75, 26$ (4C)
$\quad 9 \quad 15.5 \quad 12.5 \quad 2.25$

5. claim "over 21" (5C)

6. test stat $t_{cal} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{21.73 - 21}{2.8/\sqrt{81}} = 2.346$ (6A)

7. Critical Value: t_α (upper one tail) with $DF = n-1 = 80$ $\quad t_{0.005} = 2.6387$

t_{cal}; 0; 2.6387

we fail to reject H_a (7.A).

8. Moving Average for forcasting.
Year 2003: $F_{t+1} = \frac{Y_t + Y_{t-1} + Y_{t-k+1}}{k} = \frac{Y_4 + Y_3 + Y_2}{3} = \frac{40+45+40}{3} = 41.67$ (8.C)

9. Exponential smoothing for forcasting: $a = 0.4$, $1-a = 0.6$
$F_2 = Y_1 = 35$; $F_3 = aY_2 + (1-a)F_2 = 0.4*40 + 0.6*35 = 37$
$F_4 = aY_3 + (1-a)F_3 = 0.4*45 + 0.6*37 = 40.2$ (9.A)

10. By the defination, the wavelike pattern with gradual variation of a year would be considered as the cyclical pattern.

11. The center line $p = \frac{\#\text{ of defects}}{\text{total }\#\text{ of samples}} = \frac{140}{1000} = 0.14$ (11.B)

12. $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.14(1-0.14)}{100}} = 0.0347$ $\quad UCL = p + 3\sigma_{\bar{p}} = 0.244$ (12.D)
$LCL = p - 3\sigma_{\bar{p}} = 0.036$

13. "Different" — two-tailed test (13B)

14. Critical Value: $\pm Z_{\alpha/2} = \pm Z_{0.005} = \pm 2.58$ $\quad -2.58 \quad 2.58 \quad Z_{cal} = 2.43$ (14A)
fail to reject H_0. not enough evidence to conclude that the proportion are different.

15.

f_{ij}	e_{ij}	$(f_{ij} - e_{ij})^2/e_{ij}$
60	$(130 \times 100)/250 = 52$	$(60-52)^2/52$
39	$(130 \times 78)/250 = 40.56$	$(39-40.56)^2/40.56$
31	$(130 \times 72)/250 = 37.44$	$(31-37.44)^2/37.44$
40	$(120 \times 100)/250 = 48$	$(40-48)^2/48$
39	$(120 \times 78)/250 = 37.44$	$(39-37.44)^2/37.44$
41	$(120 \times 72)/250 = 34.56$	$(41-34.56)^2/34.56$

$\sum_i \sum_j (f_i - e_i)^2 / e_i = 4.997$ (15D)

$\chi^2_{0.01} = 9.210$ $(DF = 3-1 = 2)$

χ^2_{cv} 9.210 Fail to reject H_0 (16B)

17. $P(5.0 < \bar{x} < 5.5) = P\left(\frac{5.0-5.5}{1.2/\sqrt{36}} < Z < \frac{5.5-5.5}{1.2/\sqrt{36}}\right) = P(-2.5 < Z < 0)$
$= 0.5 - 0.0062 = 0.4938$ (17D)

0.5; 0.0062; -2.5; 0; 0.9938; 2.5

18. $P(\bar{x} > 6) = P\left(Z > \frac{6-5.5}{1.2/\sqrt{36}}\right) = P(Z > 2.5) = 1 - 0.9938$ (18C)
$= 0.0062$

19. find the value of Z (below) $Z = -1.19$ $\quad$ 0.117
$\Rightarrow -1.19 = \frac{\bar{x} - 5.5}{1.2/\sqrt{36}} \Rightarrow -1.19 * 1.2/\sqrt{36} = \bar{x} - 5.5 \Rightarrow \bar{x} = 5.262$ (19C)

20. $\bar{R} = 29.6/8 = 3.7$; $\bar{\bar{x}} = 113.2/8 = 14.15$, $A_2 = 0.729$ (20.D)
$\bar{x}$-chart $UCL = 14.15 + 0.729 * 3.7 = 16.85$, $LCL = 14.15 - 0.729 * 3.7 = 11.45$

21. largest: day 7. 17.3 > UCL = 16.85 (21.D)

22. CI for proportion p: $\frac{27}{200} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{27/200 * (1 - 27/200)}{200}}$ $100(1-\alpha)\% = 90\%$, $\alpha = 0.1$, $\alpha/2 = 0.05$, $Z_{0.05} = 1.645$

$\Rightarrow 0.135 \pm 1.645 * \sqrt{\frac{0.135(1-0.135)}{200}} = 0.135 \pm 0.040$ (22.B)

23. true proportion is between $0.135 - 0.062 = 7.3\%$ and $0.135 + 0.062 = 19.7\%$ (23B)

24. CI for mean μ (S is given) critical value $t_{\frac{\alpha}{2}} = t_{0.025}$ ($DF = n-1 = 30$) $= 2.0423$

$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 225 \pm 2.0423 * \sqrt{225}/\sqrt{31} = 225 \pm 5.50$ (24A)

25. $b_1 = 2.25$ (25.D)

26. $r^2 = SSR/SST = 0.9169 \Rightarrow 91.69\%$ of the variation in y can be explained by x. (26.A)

27. $S = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{2530.5 - 2320.3}{18}} = 3.417$, $S_{b_1} = \frac{S}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{3.417}{\sqrt{150.5}} = 0.27853$

$t_{cal} = \frac{b_1}{S_{b_1}} = \frac{2.25}{0.27853} = 8.078$ (27.B)

28. find 90% CI for $E(y)$ $\hat{y} = 30.5 + 2.25 * 20 = 75.5$, critical value $t_{0.05} = 1.7341$, $DF = 18$

CI: $75.5 \pm 1.7341 * 3.417 * \sqrt{\frac{1}{20} + \frac{(20-25)^2}{150.5}} = [72.75, 78.26]$ (28.B)

29. $b(n=5, p=0.75)$ $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - [\frac{5!}{0!(5-0)!} 0.75^0 (1-0.75)^5 + \frac{5!}{1!(5-1)!} 0.75^1 (1-.75)^{5-1}]$

$= 1 - [.25^5 + 5 * .75 * .25^4] = 0.9844$ (29.D)

30. $\mu = n \times p = 5 \times .75 = 3.75$ (30C)

31. Average —— continuous random variable (31.C)

32. $b_0 = 30.946$, $b_1 = 0.5173$, $b_2 = -0.1644$, $b_3 = -1.0965$ (32.D)

33. $x_1 = 120$, $x_2 = 10$, $x_3 = 0$ $\hat{y} = 30.946 + 0.5173 * 120 - 0.1644 * 10 = 91.38$ (33A)

34. p-value $= 0.0308 > \alpha = 0.01$, we fail to reject H_0, Time is not significant (34B)

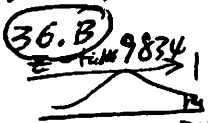
35. 95% CI: critical value: $t_{\frac{\alpha}{2}} = t_{0.025}$, $DF = 25-3-1 = 21$: $t_{\frac{\alpha}{2}} = 2.0796$ (35B)

(35B) $b_2 \pm t_{\frac{\alpha}{2}} S_{b_2} \Rightarrow -0.1644 \pm 2.0796 * 0.072 = [-0.3141, -0.147]$

36. $P(\text{Alcohol} \cup 3\text{cars}) = P(\text{Alcohol}) + P(3\text{cars}) - P(\text{Alcohol} \cap 3\text{cars})$

$= 170/400 + 50/400 - 20/400 = 200/400$ (36.B)

37. two-tailed test: p-value $= 2P(Z > |Z_{cal}|) = 2 * P(Z > 2.13)$



$= 2 * [1 - P(Z < 2.13)] = 2 * [1 - 0.9834] = 0.0332 > \alpha = 0.025$

we fail to reject H_0. (37.A)

38. Event: 70 or above: 6, total: 25 $\Rightarrow 6/25 = 0.24 \Rightarrow 24\%$ (38C)

39. Left-tail: below 20 (39C)

40. Empirical rule (40C) 95.44%

41. # of books: ratio —— natural zero (41.A)

42. $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) = 0.05 + 0.20 + 0.55 = 0.80$ (42B)

43

x	f(x)	x f(x)
0	0.10	0*0.1 = 0
1	0.10	1*0.1 = 0.1
2	0.05	2*0.05 = 0.1
3	0.20	3*0.20 = 0.6
4	0.55	4*0.55 = 2.2
		$\mu = \Sigma = 3$

(43.C)

44.

x	f(x)	$(x-\mu)^2 f(x)$
0	0.10	$(0-3)^2 * 0.1 = 9 * 0.1 = 0.9$
1	0.10	$(1-3)^2 * 0.1 = 4 * 0.1 = 0.4$
2	0.05	$(2-3)^2 * 0.05 = 1 * 0.05 = 0.05$
3	0.20	$(3-3)^2 * 0.2 = 0$
4	0.55	$(4-3)^2 * 0.55 = 1 * 0.55$
		$\sigma^2 = \Sigma = 1.9$

std.dev $\sigma = \sqrt{1.9} = 1.38$ (44.D)

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