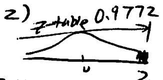
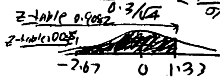
Sample EX 2

1-3. $X : N(\mu = 32.2,\ \sigma = 0.3),\ n = 4$ R.V. $\bar{x}$ (mean amount)

1. $P(\bar{x} > 32.5) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{32.5-32.2}{0.3/\sqrt{4}}\right) = P(z > 2)$
   " more than"
   $= 1 - 0.9772 = 0.0228$ (1C)



2. "In between" $P(31.8 < \bar{x} < 32.4) = P\left(\frac{31.8-32.2}{0.3/\sqrt{4}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{32.4-32.2}{0.3/\sqrt{4}}\right)$
   $= P(-2.67 < z < 1.33)$
   $= 0.9082 - 0.0038$
   $= 0.9044$ (2A)



3. "Below" Given Left-tail prob = 55.96%, find $\bar{x}$
   step 1 z ↑ 0.5596 $\quad z = 0.15$
   step 2 $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = z \Rightarrow \frac{\bar{x}-32.2}{0.3/\sqrt{4}} = 0.15 \Rightarrow \bar{x} - 32.2 = 0.15 \times 0.15$
   $\Rightarrow \bar{x} = 0.15 \times 0.15 + 32.2 = 32.223$ (3B)

4. Given $\sigma = 25$ (population standard), $n = 60$, $\bar{x} = 100$,
   set $100(1-\alpha)\% = 90\% \Rightarrow \alpha = 0.1 \Rightarrow \frac{\alpha}{2} = 0.05$, $Z_{0.05} = 1.645$
   $\bar{x} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 100 \pm 1.645 \times \frac{25}{\sqrt{60}} = [100 - 1.645 \times \frac{25}{\sqrt{60}},\ 100 + 1.645 \times \frac{25}{\sqrt{60}}]$
   $= [94.69,\ 105.31]$ (4A)

Sample EX2

5. $100 \pm 6.33 = [93.67,\ 106.33]$, we are 95% sure that the population mean (true mean) will be between 93.67 and 106.33. (5D)

6. Proportion: 90% CI. CV: $Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.645$ $\quad \bar{p} = \frac{118}{200} = 0.59$
   $\bar{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.59 \pm 1.645 \times \sqrt{\frac{0.59(1-0.59)}{200}} = 0.59 \pm 0.057$ (6C)

7. $0.59 \pm 0.068 = [0.522,\ 0.658] = [52.2\%,\ 65.8\%]$, true proportion (7A)

8. S is given use $t_{\frac{\alpha}{2}} \Rightarrow DF = n-1 = 15-1 = 14$ $\quad t_{0.05} = 1.7613$
   $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 60.50 \pm 1.7613 \times \frac{\sqrt{400}}{\sqrt{15}} = 60.50 \pm 9.10$ (8D)

9. Key word "OVER", "mean" $\quad H_0: \mu \le 21$
   claim ← $H_1: \mu > 21$ (9B)

10. S is given t-test $\quad t_{cal} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{21.73-21}{3.8/\sqrt{100}} = 1.92$ (10A)

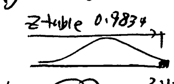
11. p-value Approach $\quad DF = n-1 = 99$

DF	0.05	0.025
99	1.6604	1.9842

   t_{cal} (between 1.6604 and 1.9842)
   p-value: $0.025 \sim 0.05 < \alpha = 0.05$
   Reject H_0
   Critical value Approach: CV: t_α (upper tail) $t_{0.05} = 1.6604$
   (11C)
   0 — 1.6604 ↗ t_{cal} $\quad$ test stat > critical value on the right, Reject H_0

12. Type I error (12B)

13. two-tailed test $\quad$ p-value $= 2P(z > |z_{cal}|)$
   $= 2P(z > 2.13) = 2 \times [1 - 0.9834]$
   $= 0.0332 < \alpha = 0.05$ $\quad$ Reject H_0 (13A)



14. Key word "has changed (different)" $\quad H_0: p_1 = p_2 \Rightarrow H_0: p_1 - p_2 = 0$ (14D)
   claim ← $H_1: p_1 \ne p_2$ $\quad H_1: p_1 - p_2 \ne 0$

15. Eg 10.16 (5D)

16. CV: two-tailed: $\pm Z_{\frac{\alpha}{2}} = \pm Z_{0.025} = \pm 1.96$ $\quad$ −1.96 — 1.96 ↗ z_{cal}
   Reject H_0, Conclude H_1. The proportion has changed (16B)

17. two-tailed test $\quad z_{cal} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{50-55}{18/\sqrt{36}} = -1.667$ (17B)

18. $\alpha = 0.01 \Rightarrow$ Critical Value $\pm Z_{\frac{\alpha}{2}} = \pm Z_{0.005} = \pm 2.58$
   −2.58 ↗ z_{cal} — 2.58 $\quad$ test stat > critical value on the left, fail to reject H_0 (18A)

19. Key "Less than" $\quad H_0: \mu_G \ge \mu_m \Rightarrow H_0: \mu_G - \mu_m \ge 0$ (19D)
   claim ← $H_1: \mu_G < \mu_m$ $\quad H_1: \mu_G - \mu_m < 0$

20. p-value Approach

DF	0.05	0.025
72	1.6663	1.9939

   $|t_{cal}|$ (between 1.6663 and 1.9939)
   p-value $= 0.025 \sim 0.05 > \alpha = 0.01$ fail to reject H_0
   Critical Value Approach $-t_\alpha = -t_{0.01} = -2.3793$ (lower tail)
   −2.3793 ↗ t_{cal} — 0 $\quad$ test stat > CV on the left fail to reject H_0 (20C)