Sample Quiz #4

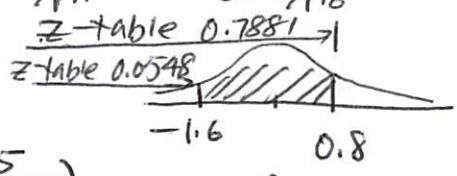
1-3: Given $X: N(\mu = 0.5, \sigma = 0.25)$; $n = 16$

keyword: "Sample mean"

1) $P(0.40 < \bar{x} < 0.55) = P\left(\frac{0.40-0.5}{0.25/\sqrt{16}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{0.55-0.5}{0.25/\sqrt{16}}\right)$

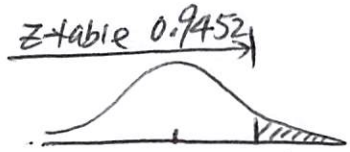
$= P(-1.6 < z < 0.8) = 0.7881 - 0.0548$

$= 0.7333$ (B)



2) $P(\bar{x} > 0.6) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{0.6-0.5}{0.25/\sqrt{16}}\right)$

$= P(z > 1.6) = 1 - 0.9452 = 0.0548$ (D)



3). step 1 find the value of z: Key "Below"

Left tail prob = 0.281

z	0.08
-0.5 ←	0.281

$z = -0.58$

step 2 use $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \Rightarrow -0.58 = \frac{\bar{x}-0.5}{0.25/\sqrt{16}}$ To find $\bar{x}$

$\Rightarrow (-0.58) \times \frac{0.25}{\sqrt{16}} + 0.5 = \bar{x} \Rightarrow \bar{x} = 0.4638 = 0.464$ (A)

4). σ is given, use $t_{\frac{\alpha}{2}}$ as the critical value

$100(1-\alpha)\% = 90\%$ $1-\alpha = .90 \Rightarrow \alpha = 0.10$, $\frac{\alpha}{2} = 0.05$

$DF = n - 1 = 30 - 1 = 29 \Rightarrow t_{\frac{\alpha}{2}} = 1.7564$ (t-table)

$\Rightarrow$ CI: $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 0.87 \pm 1.7564 \times \frac{\sqrt{0.64}}{\sqrt{30}}$

$= 0.87 \pm 0.403$ (D)

5) $\sigma = 2.5$, $n = 50$, $\bar{x} = 12.5$, find the 95% CI for μ

$100(1-\alpha)\% = 95\%$ $1-\alpha = .95$, $\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

σ is given, use $z_{\frac{\alpha}{2}}$ for the critical value

$z_{0.025} = 1.96$

CI: $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow 12.5 \pm 1.96 \times \frac{2.5}{\sqrt{50}} \Rightarrow 12.5 \pm 0.69$

$= [12.5 - 0.69, 12.5 + 0.69] = [11.81, 13.19]$ (5A)

6). $12.5 \pm 0.91 = [12.5 - 0.91, 12.5 + 0.91] = [11.59, 13.41]$ (6.C)

7). $100(1-\alpha)\% = 95\% \Rightarrow 1 - \alpha = 0.95$ $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

CI for proportion: critical value $z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

$\Rightarrow \bar{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \frac{5}{100} \pm 1.96 \times \sqrt{\frac{\frac{5}{100}(1-\frac{5}{100})}{100}}$

$= 0.05 \pm 0.043$ (7.B)

8). $0.05 \pm 0.036 = [0.014, 0.086]$ for true proportion p. (8C)

9). $\bar{x}$, $\bar{p}$ — statistic, μ, p — parameter. (9.A)