

Math 2345-Spring 2020 Business Statistics

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Office Hours: MWF 11:20 am to 11:50 pm and 2:00 pm to 3:30 pm

Teaching Assistants: See announcements for full details on TA's

WEB Site: <http://www.math.ttu.edu/~xgilliam>

Prerequisites: Math 13xx with at least grade of C. **Textbook:** “Modern Business Statistics” (6e, by Anderson, Sweeney, Williams, Camm, and Cochran)

Civility in the Classroom: Students are expected to assist in maintaining a classroom environment that is conducive to learning. In order to assure that all students have an opportunity to gain from time spent in class, troublesome behavior will not be tolerated. At a minimum, this includes using cellular phones, making offensive remarks, reading newspapers, sleeping or engaging in any other form of distraction.

Honesty: Any acts of scholastic dishonesty such as cheating, plagiarism, and collusion are not allowed.

Daily Homework, On-line Homework and Quizzes: Daily Homework will be assigned from the textbook. There will be 9 in-class multiple choice quizzes and 8 on-line Homework sets based on the daily Assignments. For the quizzes, the two lowest quiz scores will be dropped and the remaining scores will be worth 15% of the final grade. For the on-line Homework, you have up to three chances to attempt the questions and the best score will be recorded. Note that the Homework deadline is at 11:59 pm of the due dates. The Homework assignment due dates are made available well in advance. You will also be reminded of these due dates as the time approaches. There is no acceptable excuse for contacting me after the due date expecting me to reopen a set for you. This includes complaints about software problems near the due date. My advice is don't wait until the last minute to turn in the on-line Homework. The best six on-line Homework scores will be worth 10% of the final grade.

On-line Homework (MindTap):

1. Login to Blackboard (Allow pop-ups).
2. Find the course.
3. Follow the prompts to register your MindTap Course.

Tests: There will be three midterms and a **Comprehensive** final. For each test you need to bring an orange scantron sheet, pencils, and a calculator. Each of the three midterms will be worth 20% and the final exam will be worth 15%.

Make-up Exams and Quizzes: Requests for make-ups will be granted **ONLY** when accompanied by a letter from an appropriate university official, i.e. the Dean of Students. In all other cases, no make-up will be given, and the procedures outlined in this syllabus will be followed. **There are no exceptions to this rule.**

Grading Policies: Grades will be assigned based on your overall course average:

90%-100%	A	80%-89.9%	B	70%-79.9%	C
60%-69.9%	D	59.9% and below	F		

Lectures: The schedule below indicates the daily topics to be covered in class, the due dates for the dates for the exams and quizzes.

Math 2345 Calendar (Spring 2020)

Date	Material Covered	Description	Text Readings (Homework)
Jan. 15	Introduction		Ch 1 (pg27-29: #4, #6, #8, #10)
Jan. 17	Presenting Data Part 1 (Categorical)		Ch 2.1,2.4 (pg45: #2, #4)
Jan. 22	Presenting Data Part 2 (Quantitative)		Ch 2.2, 2.3 (pg62: #14, #16)
Jan.24	About on-line Homework and Quiz 1		
Jan. 27	Measure of Location and Variability		Ch 3.1, 3.2 (pg122-123: #2, #4; pg132: #24)
Jan. 29	Distribution Shapes and Boxplot		Ch 3.3, 3.4 (pg149: #46, #48)
Jan. 31	Coefficient of Correlation		
Feb. 3	Basic Probability and Conditional Probability		Ch 4.2-4.4 (pg201: #22, pg208: #32)
Feb. 5	More Probabilities and Quiz 2		
Feb. 7	Discrete Random Variable and Its Applications		Ch 5.1-3 (pg237-238: #10, #14, #16)
Feb.10	Binomial Distribution		Ch 5.5 (pg263-265: #32, #34, #42)
Feb.12	Normal Distribution		Ch 6.2 (pg303-305: #10, #12, #18, #22)
Feb.14	Normal Distribution (Cont.) and Quiz 3		Ch 6.2
Feb.17	Summary and Review for EX1		
Feb.19	Wednesday	Exam 1 (Ch1-Ch6)	
Feb.21	Sampling Distributions		Ch7.1-7.6 (pg342-343: #16, #20, #24 pg348:#28)
Feb.24	Confidence Interval for Mean		Ch 8.1-8.2 (pg371-372: #2, #10; pg381: #14, #16)
Feb.26	Confidence Interval for Proportion		Ch 8.4 (pg391: #32, #36)
Feb. 28	Intro. Hypothesis Tests and Quiz 4		Ch 9.1-9.2 (pg410: #2, #4, pg413: #8)
Mar. 2	Hypotheses Testing for a Single Mean		Ch 9.3-9.4 (pg427: #10, #12 pg435-436: #24, #26)
Mar. 4	Hypotheses Testing for a Single Proportion		Ch 9.5 (pg442: #36, #38)
Mar.6	Hypotheses Testing for two Means and Quiz 5		Ch 10.1
Mar. 9	Hypotheses Testing for two Means (cont.)		Ch 10.2-3 (pg465-467: #2,#4; pg476: #10)
Mar.11	Hypotheses Testing for two Proportions		Ch 10.3-10.4 (pg483: #20; pg493: #29, #34)
Mar.13	Some Remarks of Testing and Quiz 6		
Mar. 23	Review for EX2		
Mar. 25	Wednesday	Exam 2 (CH7-CH10)	
Mar. 27	Chi-Square Test for proportions		Ch 12.3 (pg554-555: #17, #18, #20)
Mar.30	Linear Regression Model		Ch 14.1-3 (pg631: #2; pg643: #16, #18)
Apr. 1	Testing and Interval Estimation		Ch 14.5 (pg655: #26)
Apr. 3	Interval Estimation for Linear Model		Ch 14.5 (page 661: #32, #36)
Apr. 6	Some remarks and Quiz 7		
Apr.8	Intro to Multiple Regression		Ch 15.1-3 (pg715-716: #1, #4; pg722: #14)
Apr. 10	Testing for Multiple Regression		Ch 15.4-5 (pg729: #20; pg750: #46)
Apr.15	Some remarks and Quiz 8		
Apr. 17	Intro to Xbar and R Charts		Ch 19.1-2 (pg931: #2, #4)
Apr. 20	Intro P Chart		Ch 19.2 (pg932: #8)
Apr.22	Remarks and Quiz 9		
Apr. 24	Review for EX3		
Apr. 27	Monday	Exam 3 (CH12, CH14, Ch15, and Ch19)	

Apr. 29 Intro to Time-Series Analysis
May 1 Review for the final (1)
May 4 Review for the final (2)

Ch 17 (pg826: #6)

Final Exam 12:00 MWF Section: Friday, May 8, 1:30 PM—4:00 PM, Room: Chemistry 049
1:00 MWF Section: Thursday, May 7, 1:30PM—4:00PM, Room: Chemistry 049

On-line Homework Schedules		
ASSIGNMENT # (Starting)	CHAPTERS	DUE DATES
Assignment 1 (Jan. 15)	Introduction to Using Aplia	Jan. 31
Assignment 2 (Jan. 27)	Ch 3 Numerical Measures	Feb. 7
Assignment 3 (Feb. 7)	Ch 5 Discrete Distribution	Feb. 17
Assignment 4 (Feb. 12)	Ch 6 Normal Distribution	Feb. 21
Assignment 5 (Feb. 24)	Ch 8 Confidence Interval	Mar. 6
Assignment 6 (Feb. 28)	Ch 9 Hypotheses Testing	Mar. 23
Assignment 7 (Mar. 30)	Ch14 Linear Regression	Apr. 13
Assignment 8 (Apr. 8)	Ch15 Multiple Regression	Apr. 27

Disabled Students: Please advise me of your condition and provide a letter of verification as soon as possible. I will make necessary accommodation.

Student absence for Observance of Religious Holy Day: Please advise me of your absence prior to the event and I will make necessary accommodation.

Student learning outcomes and assessment: Students will learn the meanings of, and computational procedures related to, the elementary statistical concepts used for making decisions in business and economics. In particular, students will

1. Appreciate the role of statistics in business decision making.
2. Understand the need to be wary of statistical claims, common pitfalls in sampling, and misrepresentation of conclusions.
3. Understand the meanings of various statistical measures, including the mean, median, mode, standard deviation, variance, and quartiles.
4. Become familiar with various graphical representations of data and learn to recognize misleading graphs.
5. Develop proficiency in real-world probability problems.
6. Understand the concept of a probability distribution and real-world problems, involving the binomial and normal distributions.
7. Understand and apply the Central Limit Theorem.
8. Compute and interpret confidence intervals.
9. Conduct and interpret hypothesis tests.
10. Understand linear regression models.
11. Understand the construction and use of control charts.
12. Understand the based concepts of time series.

Printing the Class Notes: Please print the class notes prior to the classes. The notes are available in PDF format on the class website through the box that says **Class Handouts:**

http://www.math.ttu.edu/~xgilliam/classes/m2345/math_2345.htm

Math 2345 EXAM # 1 Formula Sheet

Numerical Descriptive Measures:

eq3.1: Sample Mean: $\bar{X} = \frac{\sum X_i}{n}$

eq3.2: Population Mean: $\mu = \frac{\sum X_i}{N}$

eq3.5 Location of the Percentile: $L_p = \frac{p}{100}(n + 1)$

eq3.6: Interquartile Range: $Q_3 - Q_1$

eq3.7: Population Variance: $\sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$

eq3.8: Sample Variance: $S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$

eq3.9: Sample Standard Deviation:

$$S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}} = \sqrt{\text{Variance}}$$

eq3.10: Population Standard Deviation:

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{n}} = \sqrt{\text{Variance}}$$

eq3.11: Coefficient of Variation: $CV = \frac{S}{\bar{X}} * 100\%$

eq3.12: Z Score: $Z_i = \frac{X_i - \bar{X}}{S}$

Basic Probability:

eq4.1: Combination $C_n^N = \frac{N!}{n!(N - n)!}$

eq4.5: The Complement: $P(A) = 1 - P(A^c)$

eq4.6: Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

eq4.7: Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

eq4.13: If two events are independent, then $P(A \cap B) = P(A)P(B)$

Discrete Random Variable:

eq5.4: Mean: $\mu = E(X) = \sum xf(x)$

eq5.5: Variance: $\sigma^2 = \sum (X_i - \mu)^2 f(x)$

Standard Deviation: $\sigma = \sqrt{\sum (X_i - \mu)^2 f(x)}$

Binomial Distribution:

eq5.10: Combination: $\binom{n}{x} = \frac{n!}{x!(n - X)!}$

eq5.12: $f(x) = \binom{n}{x} p^x (1 - p)^{n - x}$

eq5.13: mean: $E(X) = \mu = np$

eq5.14: Variance: $\sigma^2 = np(1 - p)$

Normal Distribution:

eq6.2: Density Function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X - \mu)^2 / (2\sigma^2)}$$

eq6.3: Converting to the Standard Normal:

$$Z = \frac{X - \mu}{\sigma}$$

Math 2345 EXAM # 2 Formula Sheet

Sampling Distribution:

eq7.1: Expected Value of \bar{X} : $E(\bar{X}) = \mu$

eq7.2 Standard Deviation of \bar{X} : $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

eq7.4: Expected Value of \bar{p} : $E(\bar{p}) = p$

eq7.6: Standard Deviation of \bar{p} : $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$

Interval Estimation:

eq8.1: $100(1 - \alpha)\%$ Confidence Interval (CI) for Mean (σ known): $\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

eq8.2: $100(1 - \alpha)\%$ CI for Mean (σ unknown):

$$\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$$

eq8.6: $100(1 - \alpha)\%$ CI for Proportion:

$$\bar{p} \pm Z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Testing:

eq9.1: The test statistic for mean μ (σ known): $Z_{cal} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

eq9.2: The test statistic for Mean μ (σ unknown):

$$t_{cal} = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} \quad \text{with } (n-1) \text{ degrees of freedom.}$$

eq9.4: The test statistic for proportion p : $Z_{cal} = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

eq10.5: Test statistic for mean difference $\mu_1 - \mu_2$ (σ_1, σ_2 known): $Z_{cal} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

eq10.8: Test statistic for Mean difference $\mu_1 - \mu_2$ (σ_1, σ_2 unknown): $t_{cal} = \frac{(\bar{X}_1 - \bar{X}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

(with eq10.7: $df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{1}{n_1-1}(\frac{s_1^2}{n_1})^2 + \frac{1}{n_2-1}(\frac{s_2^2}{n_2})^2}$)

eq10.9: Test statistic for mean difference (matched samples): $t_{cal} = \frac{\bar{d} - \mu_d}{\frac{S_d}{\sqrt{n}}}$

(with $(n - 1)$ degrees of freedom)

eq10.16: Test statistic for the difference between two proportions $p_1 - p_2$:

$$Z_{cal} = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}}, \text{ where eq 10.15: } \bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2}$$

Note:

two-tailed test: $p - value = 2P(Z > |Z_{cal}|)$;

upper, one-tail test: $p - value = P(Z > Z_{cal})$;

lower, one-tail test: $p - value = P(Z < Z_{cal})$.

$Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$, $Z_{0.005} = 2.58$.

Math 2345 EXAM # 3 Formula Sheet

χ^2 Test: eq12.5: The test stat: $\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}}$, where $e_{ij} = \frac{(\text{Row } i \text{ Total})(\text{Column } j \text{ Total})}{\text{Total sample size}}$

Simple Linear Regression:

eq14.4: Estimated Simple Regression Equation: $\hat{y} = b_0 + b_1x$

eq14.6: The Slope $b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$

eq14.7: The Y-intercept: $b_0 = \bar{y} - b_1\bar{x}$

eq14.8: Sum of squares Due to Error: $SSE = \sum(y_i - \hat{y}_i)^2$

eq14.9: Total sum of squares: $SST = \sum(y_i - \bar{y})^2$

eq14.10: Sum of Squares Due to Regression: $SSR = \sum(\hat{y}_i - \bar{y})^2$

eq14.11: Relationship Among SST, SSR, and SSE: $SST = SSR + SSE$

eq14.12: Coefficient of determination: $r^2 = \frac{SSR}{SST}$

eq 14.15: Mean Square Error: $s^2 = MSE = \frac{SSE}{n-2}$

eq14.16: Standard error of the estimate: $s = \sqrt{\frac{SSE}{n-2}}$

eq14.18: Estimated Standard Deviation of b_1 : $s_{b_1} = \frac{s}{\sqrt{\sum(x_i - \bar{x})^2}}$

eq14.19: t Test Statistic $t_{cal} = \frac{b_1}{s_{b_1}}$ with $(n - 2)$ degrees of freedom

eq14.20: Mean Square Regression: $MSR = \frac{SSR}{\#indvar}$ eq14.21: F Test Statistic: $F = \frac{MSR}{MSE}$

eq14.24: $100(1 - \alpha)\%$ CI for $E(y^*)$: $\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$ where (eq14.23): $s_{\hat{y}^*} = s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$

Note: $100(1 - \alpha)\%$ CI for B_1 : $b_1 \pm t_{\alpha/2} s_{b_1}$

Multiple Linear Regression:

eq15.3: Estimated multiple regression equation: $\hat{y} = b_0 + b_1X_1 + b_2X_2 + \dots + b_pX_p$

eq15.8: Multiple Coefficient of Determination: $R^2 = \frac{SSR}{SST}$

eq15.9: Adjusted Multiple Coefficient of Determination: $R_a^2 = 1 - (1 - R^2) \frac{n - 1}{n - p - 1}$

eq15.15: t test Statistic: $t_{cal} = \frac{b_i}{s_{b_i}}$ with $(n-p-1)$ degrees of freedom.

Note: $100(1 - \alpha)\%$ CI for B_i : $b_i \pm t_{\alpha/2} s_{b_i}$

Control Charts: Xbar and R Chart

eq19.1: Standard Error of the Mean: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

\bar{x} Chart for standard deviation known:

eq19.2: $UCL = \mu + 3\sigma_{\bar{x}}$

eq 19.3: $LCL = \mu - 3\sigma_{\bar{x}}$

eq19.4: Overall Sample Mean: $\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_k}{k}$

eq19.5: Average Range: $\bar{R} = \frac{R_1 + R_2 + \dots + R_k}{k}$

eq19.8: Control limits For an \bar{x} Chart for standard deviation unknown: $\bar{\bar{x}} \pm A_2\bar{R}$

Control limits for an R chart:

eq19.14: $UCL = \bar{R}D_4$ eq19.15: $LCL = \bar{R}D_3$

Control Chart: p Chart

eq19.16: Standard Error of the Proportion: $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$

Control Limits for a p Chart:

eq19.17: $UCL = p + 3\sigma_{\bar{p}}$

eq19.18: $LCL = p - 3\sigma_{\bar{p}}$

Math 2345 Formulas After EXAM # 3

Time Series Chapter:

eq17.1: Moving Average Forecast of order k :

$$F_{t+1} = \frac{\sum(\text{most recent } k \text{ data values})}{k} = \frac{y_t + y_{t-1} + \dots + y_{t-k+1}}{k}$$

eq17.2: Exponentially Smoothed Forecast

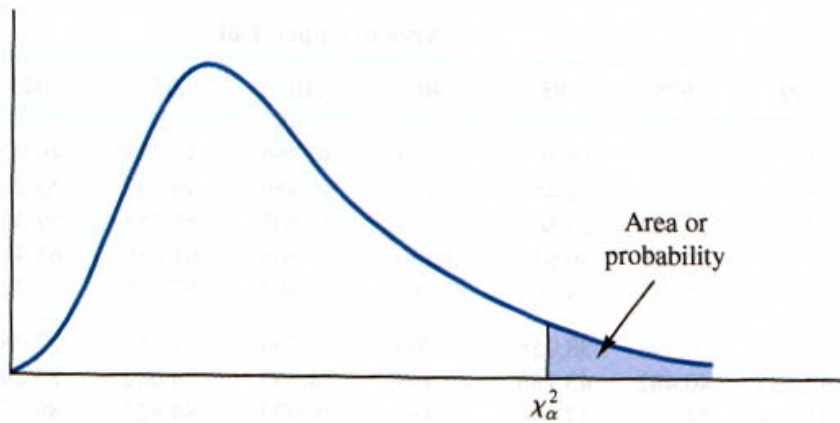
$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

TABLE 19.3 FACTORS FOR \bar{x} AND R CONTROL CHARTS

Observations in Sample, n	d_2	A_2	d_3	D_3	D_4
2	1.128	1.880	0.853	0	3.267
3	1.693	1.023	0.888	0	2.574
4	2.059	0.729	0.880	0	2.282
5	2.326	0.577	0.864	0	2.114
6	2.534	0.483	0.848	0	2.004
7	2.704	0.419	0.833	0.076	1.924
8	2.847	0.373	0.820	0.136	1.864
9	2.970	0.337	0.808	0.184	1.816
10	3.078	0.308	0.797	0.223	1.777
11	3.173	0.285	0.787	0.256	1.744
12	3.258	0.266	0.778	0.283	1.717
13	3.336	0.249	0.770	0.307	1.693
14	3.407	0.235	0.763	0.328	1.672
15	3.472	0.223	0.756	0.347	1.653
16	3.532	0.212	0.750	0.363	1.637
17	3.588	0.203	0.744	0.378	1.622
18	3.640	0.194	0.739	0.391	1.608
19	3.689	0.187	0.734	0.403	1.597
20	3.735	0.180	0.729	0.415	1.585
21	3.778	0.173	0.724	0.425	1.575
22	3.819	0.167	0.720	0.434	1.566
23	3.858	0.162	0.716	0.443	1.557
24	3.895	0.157	0.712	0.451	1.548
25	3.931	0.153	0.708	0.459	1.541

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TABLE 3 CHI-SQUARE DISTRIBUTION



Entries in the table give χ^2_{α} values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi^2_{.01} = 23.209$.

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	.000	.000	.001	.004	.016	2.706	3.841	5.024	6.635	7.879
2	.010	.020	.051	.103	.211	4.605	5.991	7.378	9.210	10.597
3	.072	.115	.216	.352	.584	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.335

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Upper-Tail Areas

Degrees of Freedom	0.25	0.10	0.05	0.025	0.01	0.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7740
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
46	0.6799	1.3022	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
48	0.6796	1.2994	1.6772	2.0106	2.4066	2.6822

continued

Upper-Tail Areas

Degrees of Freedom	0.25	0.10	0.05	0.025	0.01	0.005
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
50	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778
51	0.6793	1.2984	1.6753	2.0076	2.4017	2.6757
52	0.6792	1.2980	1.6747	2.0066	2.4002	2.6737
53	0.6791	1.2977	1.6741	2.0057	2.3988	2.6718
54	0.6791	1.2974	1.6736	2.0049	2.3974	2.6700
55	0.6790	1.2971	1.6730	2.0040	2.3961	2.6682
56	0.6789	1.2969	1.6725	2.0032	2.3948	2.6665
57	0.6788	1.2966	1.6720	2.0025	2.3936	2.6649
58	0.6787	1.2963	1.6716	2.0017	2.3924	2.6633
59	0.6787	1.2961	1.6711	2.0010	2.3912	2.6618
60	0.6786	1.2958	1.6706	2.0003	2.3901	2.6603
61	0.6785	1.2956	1.6702	1.9996	2.3890	2.6589
62	0.6785	1.2954	1.6698	1.9990	2.3880	2.6575
63	0.6784	1.2951	1.6694	1.9983	2.3870	2.6561
64	0.6783	1.2949	1.6690	1.9977	2.3860	2.6549
65	0.6783	1.2947	1.6686	1.9971	2.3851	2.6536
66	0.6782	1.2945	1.6683	1.9966	2.3842	2.6524
67	0.6782	1.2943	1.6679	1.9960	2.3833	2.6512
68	0.6781	1.2941	1.6676	1.9955	2.3824	2.6501
69	0.6781	1.2939	1.6672	1.9949	2.3816	2.6490
70	0.6780	1.2938	1.6669	1.9944	2.3808	2.6479
71	0.6780	1.2936	1.6666	1.9939	2.3800	2.6469
72	0.6779	1.2934	1.6663	1.9935	2.3793	2.6459
73	0.6779	1.2933	1.6660	1.9930	2.3785	2.6449
74	0.6778	1.2931	1.6657	1.9925	2.3778	2.6439
75	0.6778	1.2929	1.6654	1.9921	2.3771	2.6430
76	0.6777	1.2928	1.6652	1.9917	2.3764	2.6421
77	0.6777	1.2926	1.6649	1.9913	2.3758	2.6412
78	0.6776	1.2925	1.6646	1.9908	2.3751	2.6403
79	0.6776	1.2924	1.6644	1.9905	2.3745	2.6395
80	0.6776	1.2922	1.6641	1.9901	2.3739	2.6387
81	0.6775	1.2921	1.6639	1.9897	2.3733	2.6379
82	0.6775	1.2920	1.6636	1.9893	2.3727	2.6371
83	0.6775	1.2918	1.6634	1.9890	2.3721	2.6364
84	0.6774	1.2917	1.6632	1.9886	2.3716	2.6356
85	0.6774	1.2916	1.6630	1.9883	2.3710	2.6349
86	0.6774	1.2915	1.6628	1.9879	2.3705	2.6342
87	0.6773	1.2914	1.6626	1.9876	2.3700	2.6335
88	0.6773	1.2912	1.6624	1.9873	2.3695	2.6329
89	0.6773	1.2911	1.6622	1.9870	2.3690	2.6322
90	0.6772	1.2910	1.6620	1.9867	2.3685	2.6316
91	0.6772	1.2909	1.6618	1.9864	2.3680	2.6309
92	0.6772	1.2908	1.6616	1.9861	2.3676	2.6303
93	0.6771	1.2907	1.6614	1.9858	2.3671	2.6297
94	0.6771	1.2906	1.6612	1.9855	2.3667	2.6291
95	0.6771	1.2905	1.6611	1.9853	2.3662	2.6286
96	0.6771	1.2904	1.6609	1.9850	2.3658	2.6280
97	0.6770	1.2903	1.6607	1.9847	2.3654	2.6275
98	0.6770	1.2902	1.6606	1.9845	2.3650	2.6269
99	0.6770	1.2902	1.6604	1.9842	2.3646	2.6264
100	0.6770	1.2901	1.6602	1.9840	2.3642	2.6259
110	0.6767	1.2893	1.6588	1.9818	2.3607	2.6213
120	0.6765	1.2886	1.6577	1.9799	2.3578	2.6174
∞	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758