CH 8: Interval Estimation (Part 1: for mean)

1. Basic Concepts

   (A) Interval estimate (Confidence Interval): An estimate of a population parameter that provides an interval believed to contain the value of the parameter. Note: the interval estimate has the form: point estimate ± margin of error.

   (B) Confidence level: The probability of including the population parameter within the confidence interval at $100(1-\alpha)\%$. Say 95%, 99%, etc.

   (C) α is called the level of significance. In this case, it is the probability that the interval estimation procedure will generate an interval that does not contain the parameter.

   (D) Why does it work?

2. Case I: $100(1-\alpha)\%$ confidence interval estimation of the mean μ (σ known).

   (A) formula:

   $$\overline{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad \text{(eq8.1)}$$

   Note: We assume: (a) The population is normally distributed or n is large; (b) The population standard deviation σ is known. (c) $Z_{\alpha/2}$ is called the critical value and $Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is the margin of error for the estimation.

   (B) $Z_{\alpha/2}$ notation:

   $Z_{\alpha/2}$ = the right-tail (upper tail) probability $\alpha/2$ point of the standard normal; i.e., the area to the right of $Z_{\alpha/2}$ is $\alpha/2$.

   EX 1 Find the values of $Z_{\alpha/2}$ for 90%, 95% and 99%

   (1) 90%

   (2) 95%

   (3) 99%

(C) Using the formula

EX 2 The computer paper is expected to have a standard deviation of 0.02inch. 100 sheets are selected and the mean is 10.998 inches. Set up a 95% confidence interval estimates of the population mean paper length.

3. Case II: $100(1-\alpha)\%$ confidence interval estimation of the mean μ (σ unknown).

(A) Formula:

$$\overline{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \qquad \text{(eq8.2)}$$

where s is the sample standard deviation.

(B) Student's t distribution: Let $x_1, x_2, \ldots\ x_n$ be a random sample from a normal population with mean μ and standard deviation σ, then $t = \dfrac{\overline{x}-\mu}{\frac{s}{\sqrt{n}}}$ is called the t-distribution with $(n-1)$ degrees of freedom.

(C) $t_{\alpha/2}$ notation

(D) How to read the t-table:

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Part 2: Confidence Interval (CI) (cont.)

(E) How to use formula (eq 8.2)

$$\overline{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \qquad \text{(eq8.2)}$$

EX 3 Suppose that a sample of 100 sales invoices is selected from the population of sales invoices during the month and the sample mean is 110.27 and the sample variance is 838.10. Set up a 95% confidence intervals for the mean μ.

4. Case III: $100(1-\alpha)\%$ confidence interval estimation for the proportion p.

(A) We use the sample proportion $\bar{p}$ to estimate the population proportion p combined with the margin of error term.

(B) The sample proportion is defined as $\bar{p} = \frac{x}{n}$,where x is the number of elements in the sample that possess the characteristic of interest and n is the sample size.

(C) In Chapter 7 we indicated that the sampling distribution of $\bar{p}$ can be approximated by a normal distribution whenever $np \geq 5$ and $n(1-p) \geq 5$. We use $Z_{\alpha/2}$ for the critical value.

(D) Formula for the $100(1-\alpha)\%$ confidence interval for the population proportion p:

$$\bar{p} \pm Z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad \text{(eq8.6)}$$

(E) How to use (eq8.6)

Step 1: Find the sample proportion $\bar{p} = \frac{x}{n}$.

Step 2: Find the critical value $Z_{\alpha/2}$.

Step 3: Compute the confidence interval.

EX 4 A company wants to determine the frequency of occurrence of invoices error. Suppose that in a sample of 100 sales invoices, 10 contain errors. Construct a 90% confidence interval for the true proportion of error.

EX 5 Out of 268 interviewed, 83 people said that they would buy a certain product. Use a 95% confidence interval to estimate the true proportion of the customer who would buy the product.