CH 5: Discrete Probability Distributions

Part 1: Discrete Probability Distribution

1. Basic Concepts

(A) **Random Variable** (x): is a numerical description of the outcome of an experiment.

EX 1 Tossing a fair coin twice. Let x be the random variable associated with the number of heads of the experiment. List all possible outcomes for x.

(B) **Discrete Random Variable**: A random variable that may assume either a finite number of values or an infinite sequence of values.

(C) **Continuous Random Variable**: A random variable that may assume any numerical value in an interval or collection of intervals.

EX 2 Determine if the following random variable is discrete or continuous.

(1) Number of cars arriving at a tollbooth in two-hour period.

(2) Amount of time spent trying to find a parking spot on campus.

(D) **Probability distribution**: A description of how the probabilities are distributed over the values of the random variable.

(E) We usually use a table or a chart to represent the discrete probability distributions.

All Possible Variables

x	$f(x)$
x_1	$f(x_1) = P(x = x_1)$
x_2	$f(x_2) = P(x = x_2)$
$\vdots$	$\vdots$
x_N	$f(x_N) = P(x = x_N)$

Associated probabilities
$\sum f(x) = 1$
$f(x) \geq 0$

EX 1 (cont) Construct the probability distribution for the experiment with random variable x (# of heads).

(F) We can use the probability distribution table to calculate some given probabilities.

Step 1: Write the probability statement.

Step 2: Find the probability.

EX 3. Probability distribution for the number of automobiles sold during a day at a car dealer is given. Find the following probabilities:

x	$f(x)$
0	0.1
1	0.2
2	0.4
3	0.2
4	0.1

(1) x is exactly 1.
(2) x is at most 2.

(3) x is between 2 and 3 (the end points are included).

(4) x is at least 1.

2. Given the probability distribution table, compute the expectation (mean, expected value), variance and standard deviation.

$$\text{Mean:} E(x) = \mu = \sum x f(x) \qquad \text{(eq5.4)}$$

$$\text{Variance:} \sigma^2 = \sum (x-\mu)^2 f(x) \qquad \text{(eq5.5)}$$

$$\text{Standard Deviation:} \sigma = \sqrt{\sigma^2} = \sqrt{\sum (x-\mu)^2 f(x)}$$

EX 3 (Cont). Compute the mean (expected value), the variance, and the standard deviation of random variable x.

EX 4 A trip Insurance policy pays \$1000 to the customer in case of a loss due to theft. If the risk of such a loss is assured to be 1 in 200. What is a fair premium?

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Part 2: Binomial Distribution

1. Characteristics of a Binomial Distribution:

   (A) The experiment consists of a sequence of n identical trials.

   (B) Each trial is classified into one of the two outcomes (Success/Failure).

   (C) The probability of a success p is the same for each trial. The probability of a failure for each trial is $1 - p$.

   (D) The trials are independent.

   EX 5. Tossing a coin 3 times. Let us assume that getting a head is a success. This experiment is a binomial distribution.

   EX 6. Selecting random multiple choice with 10 questions, each question has 4 possible answer. This is also a binomial distribution.

2. Binomial Distribution Formula

   Given a binomial distribution with n trials and success probability p, then the probability of x successes is (called binomial probability function)

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad \text{(eq5.12)}$$

   Where: $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ (Note: $n! = n*(n-1)*(n-2)*\ldots 2*1$; $0! = 1$; $1! = 1$).

   $x =$ the number of successes in the sample ($x = 0, 1, \ldots, n$).

   $n =$ the number of trials.

   $p =$ Probability of success, $1 - p =$ Probability of failure

EX 5(cont.) Compute the probability of all possible outcomes using Eq.5.12

EX 6(cont.) Find the probability of getting exactly 6 questions right.

EX 7. A roofing contractor estimates that after the "quick fix" job on leaking roofs is done, 15% of the roofs will still leak. He fixed eight roofs, find the probability that at least two of these roofs will still leak.

3. Binomial Mean, Variance, and Standard Deviation

$$\text{Mean: } E(x) = \mu = np \tag{eq5.13}$$

$$\text{Variance: } Var(x) = \sigma^2 = np(1-p) \tag{eq5.14}$$

$$\text{Standard Deviation: } \sigma = \sqrt{np(1-p)}$$

EX 8 Suppose that past history shows that 7% of the production is defective, 200 samples are selected, find the mean, the variance, and the standard deviation of the problem.