CH 4: Basic Probability

1. Basic Concepts

   (A) **Sample Space**: The collection of all possible outcomes.

   (B) **An Event**: An event is a subset (part) of the sample space in which you are interested.

   (C) **Combination:**

$$C_n^N = \frac{N!}{n!(N-n)!} \qquad \text{(eq4.1)}$$

   $N! = N(N-1)(N-2)\cdots(2)(1)$, $n! = n(n-1)(n-2)\cdots(2)(1)$, and $1! = 1$, $0! = 1$

EX 1. From a committee of 10 people, in how many ways can we choose a subcommittee of 3 people?

   (D) **Probability:** A numerical measure of the likelihood that an event will occur.

EX 2. Two coins are tossed, find the probability of getting at least one head.

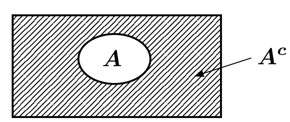
   Note 1: The probability of an event is a number between 0 and 1.

   Note 2: The probability of the sample space is 1.

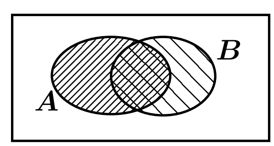
   (E) Some basic set notations and formulas.

   (1) The complement of an event A, denoted by A^c (the set of all outcomes that are not in A). The equation for computing probability using the complement is given by

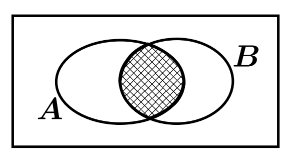
$$P(A) = 1 - P(A^c) \qquad \text{(eq4.5)}$$



   (2) The union of two events A, B, denoted by $(A \cup B)$, is the set of all outcomes that are in A, B or both.



   (3) The intersection of two events A, B, denoted by $(A \cap B)$, is the set of all outcomes that are in A and B. Note: $P(A \cap B)$ is the joint probability.



   (4) If the intersection $(A \cap B)$ is empty (i.e. $P(A \cap B) = 0$), then the two events A, B are called mutually exclusive (disjoint).

(5) Addition law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B). \quad \text{(eq4.6)}$$

(F) Conditional Probability:

The probability of the occurrence of an event A, given the occurrence of another event B, denoted by $P(A|B)$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}. \quad \text{(eq4.7)}$$

Similarly, the conditional probability of event B given that event A has occurred as

$$P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

Now, based on the conditional probability, we can write $P(A \cap B) = P(A|B)P(B)$ or $P(A \cap B) = P(B|A)P(A)$ (called the multiplication law). If $P(A|B) = P(A)$ and $P(B|A) = P(B)$, then A and B are said to be independent.

(G) Independent Events: Two events A and B are said to be statistically independent if and only if

$$P(A \cap B) = P(A)P(B). \quad \text{(eq4.13)}$$

EX 3. Suppose that an employment agency has found that 40% of the applicants are college grads and 30% of the applicants have had computer skills. The two characteristics are independent. Find the probability that the applicants are college grads and have had computer skills.

2. Application: Computing Probability from crosstabulations.

	A	B	C	Total
D				
E				
Total				

EX 4. The manager of a shirt manufacture wants to study the connection between shifts and shirt quality. 600 shirts are randomly selected. The results are shown below:

Shirt quality	Shift 1	Shift2	Shift3	Total
Perfect	240	191	139	570
Flawed	10	9	11	30
Total	250	200	150	600

(1) What proportion (%, probability) of the shirts were perfect or made by shift 2?

(2) Given that the shirts were made by shift 2, what proportion (%, probability) of the shirts were perfect?