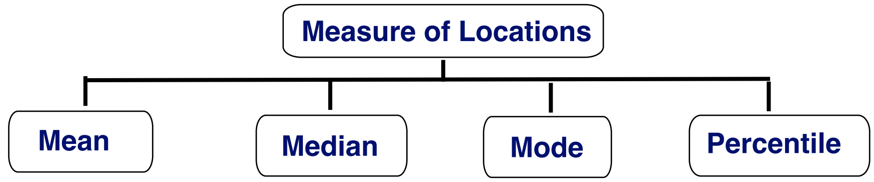
CH 3: Descriptive Statistics: Numerical Measures Part 1



1. Measure of Locations

   (A) Observation Notation x_i: the ith observation in the list of observations.

   (B) Summation Notation Σ ("Sigma"–Computing the sum):

   We write $\Sigma_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$

   (C) Sample Mean (Notation: $\bar{x}$)

$$\bar{x} = \frac{\Sigma x_i}{n} \qquad \text{(eq3.1)}$$

EX 1 Given a set of data with $n = 5$ (the birth weights): 9.2, 6.4,10.5, 8.1,7.8. Find the mean.

   (D) The Population Mean (Notation: μ)

$$\mu = \frac{\Sigma x_i}{N} \qquad \text{(eq3.2)}$$

   (E) Median: the middle value when the observations are arranged in ascending order (smallest value to largest value).

   Note 1: For an odd number of observations, the median is the middle value; for an even number of observations, the median is the average of the two middle values.

EX 1 (cont.) Find the median.

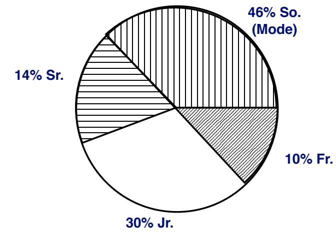
EX 2 Find the mean and median of the data set: ($n = 6$) 15, 3, 46, 623, 126, 64, Find the mean and the median.

Note 2: In some cases, median is a more sensible measure of center than the mean, for example, government uses median income.

(F) Mode: The mode is the value that occurs with greatest frequency.

EX 3 Find the mode for the following ordered array: 0, 0, 1, 2, 2, 3, 3, 3, 3, 3, 4, 5, 6, 26.

EX 4 Find the mode for the pie chart.



(G) Percentile: The pth percentile is a value such that at least p percent of the observation are less than or equal to this value and at least $(100 - p)$ percent of the observations are greater than or equal to the value. To find the percentile, the following procedure can be used:

(1) Order the data from the smallest to the largest.

(2) Find the location of the pth percentile

$$L_p = \frac{p}{100}(n+1) \tag{eq3.5}$$

(3) Rules to follow: ithe rank is split into integer component k and decimal component d, such that $L_p = k + d$. The value (the pth percentile) is calculated as

$$r_k + d(r_{k+1} - r_k)$$

EX 5 Given a set of data: 15, 20, 25, 25, 27, 28, 30, 34. Find the 20th percentile and the 75th percentile.

2. Measures of Variability

(A) Variance (Notation: Sample Variance S^2, Population Variance σ^2)

$$\sigma^2 = \frac{\sum(x_i - \mu)^2}{N} \tag{eq3.7}$$

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} \tag{eq3.8}$$

(B) Standard Deviation ((Notation: Sample Variance s, Population Variance σ)

$$s = \sqrt{s^2} \tag{eq3.9}$$

$$\sigma = \sqrt{\sigma^2} \tag{eq3.10}$$

EX 6 Given a set of data: $n = 5$: 3, 7, 5, 8, 7. Find the variance and the standard deviation.

Step 1: Find

$$\bar{x} = \frac{1}{5}\sum_{i=1}^{5} x_i = \frac{3+7+5+8+7}{5} = 6$$

Step 2: Set up a table to find $(x - \bar{x})^2$

obs.	$(x_i - 6)^2$
3	$(3-6)^2 = 9$
7	$(7-6)^2 = 1$
5	$(5-6)^2 = 1$
8	$(8-6)^2 = 4$
7	$(7-6)^2 = 1$

$\Rightarrow \Sigma = 16$

Step 3: Sample Variance

$$S^2 = \frac{16}{5-1} = 4$$

Step 4: Standard Deviation

$$S = \sqrt{4} = 2$$

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(C) Range

$$\text{Range} = \text{Largest value} - \text{Smallest value}$$

EX 6 cont. Find the range of the data set: ($n = 5$: 3, 7, 5, 8, 7).

(D) Interquartile Range

$$\text{Interquartile Range} = Q_3 - Q_1 \qquad \text{(eq3.6)}$$

(1) Quartiles: dividing the ordered data into four portions.
(2) Q_1: the first quartile (25th percentile).
(3) Q_2: the second quartile (the median, 50th percentile).
(4) Q_3: the third quartile (the 75th percentile).

EX 5 (cont.) Given a set of data: 15, 20, 25, 25, 27, 28, 30, 34. Find Q_1, median(Q_2), and Q_3 and find the interquartile range.

(E) Coefficient of Variation

$$\left(\frac{\text{Standard deviation}}{Mean} \times 100\right)\% = \frac{s}{\bar{x}} \times 100\% \qquad \text{(eq3.11)}$$

CV is used in comparing two or more sets of data measured in different units

3. Five Number Summary and the Boxplot

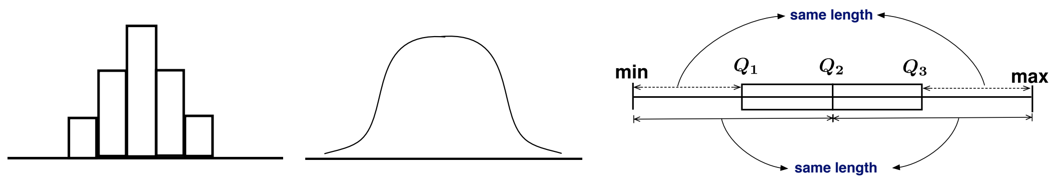
(A) The five-number summary: smallest value, Q_1, Q_2(median), Q_3, largest value

(B) Boxplot: A graphic display of the Five-Number Summary

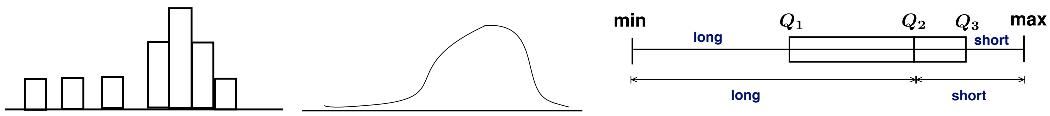
EX 5 (cont.) Construct the Boxplot of the given data set.

(C) Distribution Shape based on Boxplot:

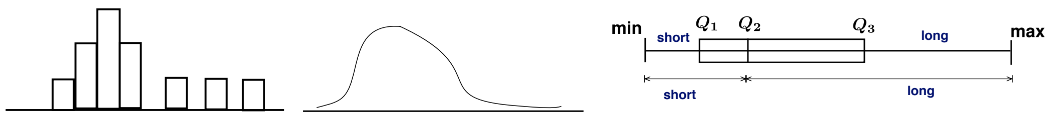
Case 1: Symmetric



Case 2: Left-skewed



Case 3: Right-skewed



EX 5 (cont.) Find the distribution shape of the data set.

Note: An important numerical measure of the shape of a distribution is called Skewness. Case 1 symmetric ($skewness = 0$);
Case 2 Left-skewed ($skewness < 0$);
Case 3 Right-skewness ($skewness > 0$)

4. z Scores

(1) z-Score

$$z_i = \frac{x_i - \bar{x}}{s} \qquad \text{(eq3.12)}$$

(2) z-score is often called the standardized value.

(3) A z-score reflects how many standard deviations above or below the population mean an observation is. For instance, on a scale that has a mean of 500 and a standard deviation of 100, a value of 450 would equal a z score of (450-500)/100 = -50/100 = -0.50, which indicates that the value is half a standard deviation below the mean.

5. The Empirical Rule:

For a "Bell-Shaped" normal distribution. About 68% (2/3 of the data) lie within one standard deviation of the mean; about 95% of the data lie within two standard deviation of the mean; Almost all (about 99.7%) of the data lie within three standard deviation of the mean.

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6. Measures of Association Between Two Variables

(A) Scatter diagram: Given paired observations (x_i, y_i) (i.e. data set that is concerning with two measurement variables x and y), a scatter diagram uses the x and y axis to represent the data.

(B) The Covariance:

(1) The covariance measures the strength of the linear relationship between two numerical variables (x and y).

(2) The sample covariance is computed from the following equation:

$$s_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

(C) The correlation Coefficient

(1) The correlation coefficient measures the strength of the linear relationship between two numerical variables (x and y).

(2) The sample correlation coefficient is computed from the following equation:

$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

where s_x is the sample standard deviation of x and s_y is the sample standard deviation of y.

(3) In particular, $-1 \leq r_{xy} \leq 1$.

EX 7 Given a set of paired observations with $n = 4$: (2 , 5), (1, 3), (5, 6), (0, 2)

(1) Obtain the scatter diagram.

(2) Compute the covariance s_{xy}.

(3) Compute the sample standard deviations s_x and s_y.

(4) compute the correlation coefficient r_{xy}.

(5) Interpret the result.