

CH 12: Testing the Equality of Population Proportions for Three or more Population Proportions

(A) The pair of hypothesis:

$$H_0: p_1 = p_2 = p_3 = \dots = p_k.$$

H_1 : Not all population proportions are equal.

(B) Test Statistic (χ^2 -test):

eq12.5: The test stat:
$$\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}}$$

(where f_{ij} is the observed frequency for the cell in row i and column j , e_{ij} expected frequency for the cell in row i and column j under the assumption H_0 is true.)

Note1: Select a random sample from each of the populations and record the observed frequencies, f_{ij} in a table with 2 rows and k columns.

Note2: The expected frequencies
$$e_{ij} = \frac{(\text{Row } i \text{ Total})(\text{Column } j \text{ Total})}{\text{Total sample size}}$$

Note 3: We set up a table to compute the test statistic

(C) How to use the χ^2 table:

(1) The test statistic has chi-square distribution with $k - 1$ degrees of freedom.

(2) α is the level of significance (upper tail).

(D) We can use either the p -value approach or the critical value approach to make a decision.

EX Suppose that in a particular study we want to compare the customer loyalty for three automobiles. Chevrolet Impala, Ford Fusion, and Honda Accord. The Hypotheses are stated as follows:

$$H_0: p_1 = p_2 = p_3$$

H_1 : Not all population proportions are equal

Sample results of likely to repurchase for three populations of automobile owners are given from the following table:

Find the test statistic and use $\alpha = 0.01$ to make a decision.

(1) Set up a table to find the test statistic: eq12.5: $\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}}$

(2) Use the p -value approach to make a decision.

(3) Use the critical value approach to make a decision.