A New Method for the Computation of Motion from Image Sequences

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***Abstract*— The object of this paper is to introduce a new method for computing the linear velocity and angular velocity of an unmanned air vehicle (UAV) using only the information obtained from image sequences. In UAV applications, computational resources are limited due to payload constraints and the real-time computation requirement. Therefore, computationally intensive techniques employing feature extraction cannot be used. The alternative, in existing literature, is the computation of optical flow and the subsequent computation of motion. Both of these problems are ill-posed due to the correspondence and aperture problems.**

In this paper, we consider a different approach for motion estimation that is based on the spatial differentiation of an image function. We show that the solution is a well-posed problem that involves a least squares problem and nonlinear filtering. We also discuss the implementation of such a scheme on a UAV, and discuss the existence of such schemes in insects and crustaceans.

I. Introduction

Compact and Low cost Inertial Navigation Systems (INS) employing MEMS acceleration sensors and piezoelectric gyroscopes have become available in recent years for use in UAV applications. For micro-UAV's operating in urban environments, such INS systems have to be used in conjunction with cameras or other sensors for collision avoidance and navigation. For such applications, it is highly desirable to reduce the payload by using the cameras for inertial measurement as well. The measurement of self-motion from images is a well studied problem in Robotics and Computer Vision [11], [23]. The techniques employed differ primarily on whether features in the image are identified or not. Methods based on former rather than the latter need more computational power. Micro-UAV's with limited payload and the need for real-time computation have a constraint on computational resources. Hence, motion estimation without feature extraction is to be considered for such applications. In particular, our objective is to study the feasibility of replacing linear acceleration and angular velocity signals from an INS with linear velocity and angular velocity signals from an *Optical Navigation System* (ONS).

Though different techniques are available in existing literature for self-motion estimation, they are all inherently based on a single camera/imaging device model - the pinhole camera. On the other hand, nature employs various imaging methods (hardware) and neural computational pathways (software) for relative motion determination, from narrow-band photosensitive pigments and filtering in deep-sea creatures [6], [8], [9], to several types of compound eyes in insects and crustaceans [20], [27] to binocular vision in mammals. In this paper, we propose a solution to the self-motion estimation problem based on the spatial differentiation of the image formed by an imaging device. For a practical implementation of this method, it will be apparent that the imaging device must correspond to any of the (apposition or superposition) compound eyes [27].

There are some theoretical advantages to the use of an ONS over an INS, once the implementation details are ironed out. An INS yields noisy estimates of the angular velocity and the linear acceleration of a UAV. The integration of the linear acceleration to obtain the linear velocity results in a random walk and the variance of the noise associated with the problem increases with time, unless it can be corrected by some external means. The ONS on the other hand yields noisy estimates of both the angular velocity and linear velocity of a UAV. Thus it is much more suitable for navigation provided the implementation details are sorted out.

In Section II, we discuss existing techniques for motion estimation that are based on a pin-hole camera. In Section III, we discuss how we have approached motion detection by describing what we call an optical navigation system (ONS). The mathematical arguments we present naturally require the use of a filtering scheme. In a related paper, we will study the use of the intrinsic observer for Lagrangian systems [3] for this purpose.

II. Existing Motion Detection Methods using a Pinhole Camera

When a pinhole camera is used to form an image on a retinal/image plane with local co-ordinates (x, y), the intensity of the image formed $I_{(x,y)}$ depends on the reflectivity of the surfaces being imaged, and the direction of the light source. In a purely descriptive sense, *optical flow* is the apparent motion of brightness patterns observed when a camera is moving relative to the objects being imaged [11]. The computation of optical flow without extracting features depends on correlating areas of pixels once the motion has taken place. As one can easily foresee, this task is ill-posed because objects get larger when the camera moves towards them. This is known as the *correspondence problem* in active vision. There are other problems that make the computation of the optical flow ill-posed. These are studied rather well by Faugeras [11].

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Below, we make a brief survey of the rather vast literature on optical flow to illustrate the difficulty involved in real-time implementation. In existing methods, one has to solve a constrained least squares optimization problem at every instant of time to obtain the optical flow [2], [5], [17], [18], [19], though not all authors agree that the *Horn constraint equation* that is used to constrain the solutions is correct [11]. Many authors begin with this constraint and attempt to solve the problem from there. One class of methods is gradient-based, and according to Beauchemin and Barron [5], many researchers have used this optical flow constraint with other constraints to compute optical flow using gradients [4], [7], [10], [14], [13], [18], [24], [25], [26], [29], [30]. One popular method is due to Horn and Schunck [18]. The optical flow constraint equation by itself is certainly not sufficient to solve for optical flow, so they introduce a smoothness constraint. Another area of optical flow computation is frequency-based. These techniques use velocity-tuned filters in the Fourier domain of time varying images. Beauchemin and Barron [5] summarize the advantages of these methods, describing different types of filtering and some hierarchical approaches, citing several researchers who have explored these techniques [1], [12], [15], [16], [33]. Another technique due to Srinivasan [31] is a non-feature-tracking technique in which parameters of motion are computed using a single-stage, non-iterative process. This process interpolates the position of images that are moving with respect to a collection of reference images. From this discussion, the reader can appreciate the difficulties in doing the optical flow computation in real-time. Liu, Hong, Herman, Camus and Chellappa [21] did a computational comparison of the available methods and studied the feasibility for real-time implementation, and their results were not promising.

Assuming that the optical flow has been computed, the second problem is the computation of the linear and angular velocities of the camera. If the pinhole camera undergoes a motion tangential to the image plane, then objects closer to the camera will have a greater displacement in the image plane than objects farther away. Due to this, one cannot make a connection between the optical flow on the retinal plane and the actual motion of points in the three dimensional world without precisely knowing the distances of some points from the camera. In a real-time application on a UAV, this involves either limited feature extraction, or one has to use additional means of distance estimation. Otherwise, one can only estimate the velocities up to an unknown scaling constant.

III. Optical Navigation System

In this section, we introduce a new approach to computing motion using a sequence of images. Unlike optical flow theory, that is based on post-processing of the image data to find correlations or correspondences in the image intensities, our theory is based on the spatial derivative of the image. For implementation of this method, an imaging device rather like a compound eye (apposition or superposition type) has to be used. This is so that one can relate the spatial derivative to the change in the intensities actually produced by motion. Then the solution of the motion parameters (linear and angular velocities of the camera) can be cast as the solution of an linear well-posed inverse problem, under some mild technical conditions on the image.

A. Mathematical Background

Let H be an infinite dimensional Hilbert space. Given $y \in H$, suppose that we wish to determine $\beta \in \mathbb{R}^m$ that satisfies

$$y = W\beta + \varepsilon, \tag{1}$$

where W is a linear operator, and $\|y - W\beta\|$ is a minimum. The solution is the well-known pseudo-inverse [22].

Theorem 3.1: Let W^* be the adjoint of W. The vector $\hat{\beta} \in \mathbb{R}^m$ minimizes $\|y - W\beta\|$ if and only if $W^*W\hat{\beta} = W^*y$. If W has full rank, the solution is $\hat{\beta} = (W^*W)^{-1} W^*y$.

Now consider the linear operator $W : \mathbb{R}^m \to L^2(U)$, where $U \subset \mathbb{R}^2$ defined by the map $\beta \mapsto \phi(x,y)$ with

$$\phi(x,y) = A(x,y)\beta\,;\ (x,y) \in U \tag{2}$$

where $A(x,y)$ is a $1 \times m$ row vector. The adjoint of W is the operator $W^* : L^2(U) \to \mathbb{R}^m$ and is defined by:

$$\langle W^*\phi, \beta\rangle_{\mathbb{R}^m} = \langle \phi, W\beta\rangle_{L^2(U)}\,,\ \beta \in \mathbb{R}^m,\ \phi \in L^2(U) \tag{3}$$

where

$$\langle u, v\rangle_{\mathbb{R}^m} = u^T v; \quad u, v \in \mathbb{R}^m, \tag{4}$$

and

$$\langle f, g\rangle_{L^2(U)} = \iint\limits_U f(x,y)g(x,y)\,dxdy;\ f, g \in L^2(U). \tag{5}$$

Using the definitions in Equations (4) and (5), we obtain from equation (3):

$$W^*\phi = \iint\limits_U A^T(x,y)\phi(x,y)\,dxdy \tag{6}$$

Then according to Theorem 3.1, the normal equation is

$$\left[\iint\limits_U A^T(x,y)A(x,y)\,dxdy\right]\beta = \iint\limits_U A^T(x,y)\phi(x,y)\,dxdy, \tag{7}$$

and the solution to equation (7) is (when $\mathrm{Range}(W) = \mathbb{R}^n$):

$$\widehat{\beta} = \left[\iint\limits_U A^T(x,y)A(x,y)\,dxdy\right]^{-1}\iint\limits_U A^T(x,y)\phi(x,y)\,dxdy. \tag{8}$$

B. ONS Theory

As we have already mentioned, previous approaches to this problem involved the solution of two ill-posed problems:

1) computation of optical flow
2) linear velocity and angular velocity computation.

The approach we choose to take involves setting up the linear equation

$$\delta I(x,y,t) = W(x,y,t)\beta + \varepsilon_t, \tag{9}$$

where $\delta I(x,y,t)$ is the observed change in image intensity at time t at the point (x,y) in the image plane with respect to a coordinate system fixed to the image plane, ε_t is a white noise process, $W(x,y,t)$ is computed from the observed image at time t, and β is the vector of unknown linear and angular velocities at time t. We then solve for β using Theorem 3.1 and use a non-linear observer to filter the noise.

Suppose that we have N imaging devices fixed to a UAV. We will call them "cameras" for brevity even though it will be clear that we do not have pinhole cameras in mind. Let the position of the center of mass of the UAV with respect to an inertial frame be given by $b(t)$ at time t, and the orientation of the principal axes be given by $Q(t) \in SO(3)$. The pair $(Q,b)(t) \in SE(3)$ denote the configuration variables for the UAV. Let the angular velocity and linear velocity of the UAV in the inertial frame be $\omega(t)$ and $v(t)$ respectively. The reason for using velocities in the inertial frame rather than in the usual body frame will be apparent in a moment. If (Ω, ξ) are the velocities in the body frame, then the usual relations are: $\omega = Q\,\Omega$ and $v = Q\,\xi$. The retinal planes of the cameras are assumed to be surfaces, and there is a family of embeddings $\mathcal{E}^i_{(Q,b)} : U_i \subset \mathbb{R}^2 \to \mathbb{R}^3$, where $i \in \{1, \cdots, N\}$, that map the local co-ordinates of a point on a retinal plane to its inertial co-ordinates.

We define the *image* functions $I_i(x,y,t)$ below and state the space in which they belong later. The x and y above are local coordinates on the image plane. The image functions are functions of the configuration variables (Q,b) of the UAV:

$$\begin{array}{rrcll} I_i : & SE(3) & \to & H^1(U_i), & i = 1, \cdots, N \\ & (Q,b) & \mapsto & I_i(Q,b,\cdot,\cdot,t) & \end{array} \tag{10}$$

The tangent maps of these functions are then defined by:

$$\begin{array}{rrcl} TI_i : & TSE(3) & \to & L^2(U_i), \\ & (Q,b,\omega,v) & \mapsto & W_i(Q,b,\cdot,\cdot,t,\omega,v) \end{array} \tag{11}$$

where $W_i(Q,b,x,y,t,\omega,v) = D_{(Q,b)}I_i(Q,b,x,y,t)(\omega,v)$.

The intensity of a pixel can change due to the motion of the pixel in directions tangent and normal to itself. We split the change in the intensity due to the motion of the pixel into the tangential and normal components. The reason for this is that we can theoretically estimate the intensity change in the tangential direction but not in the normal direction.

Now consider the point $(x,y) \in \mathbb{R}^2$ on a image plane that has coordinates (X,Y,Z) in the inertial frame. The point (X,Y,Z) has the velocity $w \in \mathbb{R}^3$ due to the combined rotational and linear motion of the UAV. In differential geometric terms, this velocity is a section $\sigma : \mathbb{R}^3 \to T\mathbb{R}^3$. Due to the embeddings $\mathcal{E}^i_{(Q,b)}$, we can pullback this section to the set U_i to which (x,y) belongs. Thus to the point $(x,y) \in U_i$ we have the velocity vector $w = \sigma \circ \mathcal{E}^i_{(Q,b)}(x,y)$. We also have the relation

$$\left(D_{(Q,b)}(X,Y,Z)\right)(\omega,v) = w.$$

At the point (X,Y,Z) in the inertial frame, let E_1 and E_2 be two vectors (in the inertial frame) that are tangent to the image plane at the point, and let the vector M be normal to the image plane. Let the vector w have the representation (w_1, w_2, w_3) in the frame (E_1, E_2, M). Now:

$$w_{1i} = (\omega \times r_i + v) \cdot E_1 \tag{12}$$
$$w_{2i} = (\omega \times r_i + v) \cdot E_2, \tag{13}$$

where $r_i \in \mathbb{R}^3$ is the vector from the center of mass of the aircraft to the center of the i^{th} image plane. Note that $(w_1, w_2)_i$ is a linear deterministic function of (ω, v). We then split the directional derivative;

$$\begin{aligned} \frac{dI_i}{dt} &= \left[D_{(Q,b)}I_i(x,y,t)\right](\omega,v) + \frac{\partial I_i}{\partial t} && (14) \\ &= \left(W_i(Q,b,x,y)\right)(\omega,v) + \frac{\partial I_i}{\partial t} && (15) \\ &= \left(D_{(X,Y,Z)}I_i\right) w + \frac{\partial I_i}{\partial t} \ \text{(Chain Rule)} \\ &= \left(D_{(x,y)}I_i\right) \cdot (w_1, w_2)_i + \varepsilon(x,y,t), \end{aligned}$$

where $\varepsilon(x,y,t)$ is the component of the change in the image intensity due to motion normal to the image plane and changes in ambient light.

The problem that we need to solve can be simply stated. Given an observed change in image intensity $\delta I_i(x,y,t)$ in time δt, we have:

$$\frac{\delta I_i}{\delta t} = \left(D_{(x,y)}I_i\right) \cdot (w_1, w_2)_i + \varepsilon(x,y,t) \tag{16}$$

and compute the best approximation $(\widehat{w}_1, \widehat{w}_2)_i$ to $(w_1, w_2)_i$ that minimizes $\left\| \frac{\delta I_i}{\delta t} - \left(D_{(x,y)}I_i\right) \cdot (w_1, w_2)_i \right\|_{L^2(U)}$, where "best approximation" means the minimization is performed in the least squares sense. The normal equation for this problem is:

$$\begin{aligned} &\left[\iint_U \left(D^T_{(x,y)}I_i\right)\left(D_{(x,y)}I_i\right)\,dxdy\right](w_1,w_2)_i \quad = \\ &\iint_U \left(D^T_{(x,y)}I_i\right) \tfrac{\delta I_i}{\delta t}(x,y,t)\,dxdy \quad + \\ &\iint_U \left(D^T_{(x,y)}I_i\right) \varepsilon(x,y,t)\,dxdy. \end{aligned} \tag{17}$$

Note here that if $N = 3$ and the left hand side of equation (17) has rank two for each i, then we have a well-posed problem for (ω, v). In other words, if $I_i(Q,b,t)$ is a plane in $\mathbb{R}^3$, then $\max\ \text{rank}\ W_i = 2$. Then we would need three planes pointing in mutually orthogonal directions. However, it is not necessary to require this rank condition for all t, but only for almost every t. It is also necessary to make a few additional assumptions:

- $I_i \in H^1(U)$ for almost every t; i.e., I_i has a square-integrable derivative.
- There does not exist $G \neq e \in SE(2) \ni I_i(G(x,y)) = I_i(x,y,t)$ for almost every t. This says that the *aperture problem* is allowed to occur over a set of measure zero in time.
- $\sigma_t = \iint_U W^T(x,y)\varepsilon(x,y,t)\,dxdy$ is a white noise process.
- The camera's frame rate is strictly greater than its maximum angular velocity. Otherwise, it is not possible to avoid the aperture problem for strictly rotational motion of the UAV.

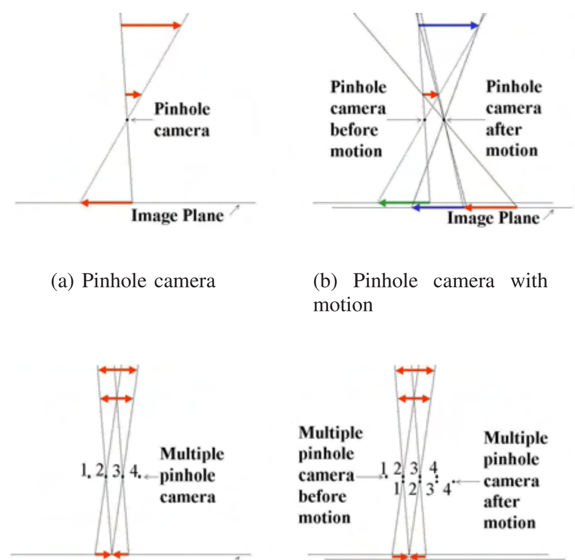
After (ω, v) is computed in this manner, they are then used as an input to the non-linear observer described by Aghannan and Rouchon [3].

C. Implementation

It is clear that for the success of the method in the previous subsection, we need to be able to compute the map $W_i(x,y) = D_{(x,y)} I_i$ and it must yield the change in the intensity at the point (x,y) on the image plane, when an actual motion (ω, v) happens. For this to happen, the images at the neighboring pixels must be produced by independent pinholes.

In Figure 1a, we have a pinhole camera. Clearly, both the near and distant object will produce the same image on the image plane. Figure 1b shows a similar situation with a pinhole camera that has undergone a small motion to the right. One should note here that even though the shifted image plane is drawn slightly below the original image plane, the pinhole camera has not moved down, only to the right. We can easily see that the near object has produced an image that moves much more than does the image from the distant object in the image plane. Because of this, one cannot determine how far the camera has moved to the right, or how fast it did so, without knowing the depth of the objects. A similar conclusion was arrived by Polat and Pachter in their investigation of INS aiding using a camera [28].

Now let's consider a multiple pinhole camera as shown in Figure 1c. Each pinhole is now responsible for producing an image on a much smaller section of the image plane, resulting in a narrower field of vision. Again, both the near and distant object create the same image on the image plane. We now consider Figure 1d. In this figure we show the camera and object from Figure 1c with a small displacement to the right. After the motion, both objects again produce the same image on the image plane, but more importantly the light producing these images is now passing through camera holes 1 and 2, not 2 and 3. One can now see that having a multiple pinhole camera allows each individual pinhole to have a narrow field of vision, aiding in the computation of motion without any knowledge of depth. The camera shown in Figures 1c and 1d is similar to the compound eye structure [20], [27] observed in insects and crustaceans. Both apposition and superposition type compound eyes have this property. For example, consider Figure 2 and Figure 3 taken from [32]. Figure 2 shows an illustration of a simple apposition eye and how each simple eye accepts light from a narrow field of vision. Light that does not come from a nearly straight on angle will be absorbed. Figure 3 shows the compound eyes of a common fly.



(a) Pinhole camera

(b) Pinhole camera with motion

(c) Multiple pinhole camera

(d) Multiple pinhole camera with motion

Fig. 1. Comparison of Pinhole and Multiple-Hole Cameras

IV. Conclusion

Existing methods for the computation of motion use the pinhole camera model and result in the formulation of two ill-posed problems. In this paper, we have described an Optical Navigation Scheme ONS that reduces the computation of the angular and linear velocities of a UAV to a linear least squares problem for each instant of time. Our formulation naturally uses a filtering idea for the computed velocities, that allows us to relax some of the conditions on the image functions. We plan to test this theory in the near future experimentally and compare the results with those obtained from an INS.

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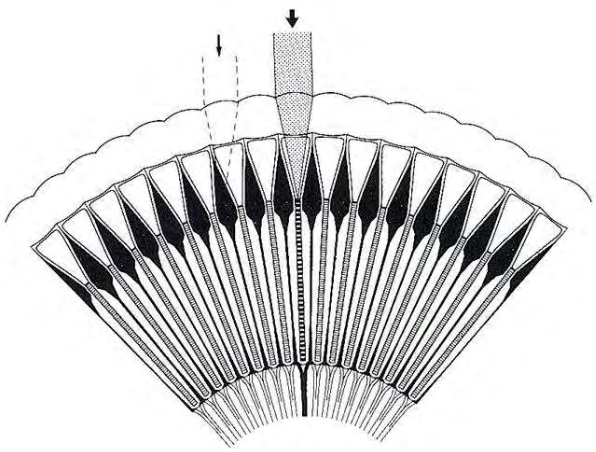

Fig. 2. Simple apposition eye [32]

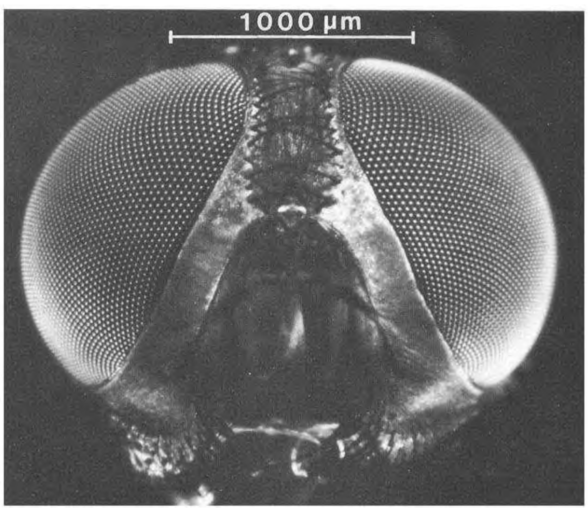

Fig. 3. Head of the fly *Musca domestica* [32]

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