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On a low-dimensional model for magnetostriction

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Abstract

In recent years, a low-dimensional model for thin magnetostrictive actuators that incorporated magneto-elastic coupling, inertial and damping effects, ferromagnetic hysteresis and classical eddy current losses was developed using energy–balance principles by Venkataraman and Krishnaprasad. This model, with the classical Preisach operator representing the hysteretic constitutive relation between the magnetic field and magnetization in the axial direction, proved to be very successful in capturing dynamic hysteresis effects with electrical inputs in the 0–50 Hz range and constant mechanical loading. However, it is well known that for soft ferromagnetic materials there exist excess losses in addition to the classical eddy current losses. In this work, we propose to extend the above mentioned model for a magnetostrictive rod actuator by including excess losses via a nonlinear resistive element in the actuator circuit. We then show existence and uniqueness of solutions for the proposed model for electrical voltage input in the space $L^2(0,T)\cap L^\infty(0,T)$ and mechanical force input in the space $L^2(0,T)$.

Keywords: Preisach operator; Magnetostriction; Eddy current losses; Low-dimensional model

1. Introduction

Smart materials like piezoelectrics and magnetostrictives have complex electro-magneto-visco-elastic constitutive relationships that give rise to rate-dependent hysteretic responses. Among them, magnetostrictive actuators show considerably more complex responses due to the presence of microscopic eddy-currents in both the magnetostrictive actuator and its ferromagnetic casing. These eddy-currents can be significant even when the frequency of the electrical excitation is as low as 10 Hz [1].

In early works [1,2], we have addressed the importance of the eddy-current modeling using energy-balance ideas. The result was a model that could be represented in the block diagram form in Figs. 1 and 2 when the resistor R_{excess} is neglected. Fig. 1 shows a three branch circuit. It models a magnetostrictive actuator connected to a voltage supply $u(\cdot)$, with a lead resistor R. The hysteretic inductor shown in one branch is an ideal inductor that accounts only for hysteresis losses. In the other two branches, the classical eddy current and excess losses are introduced by using the resistors $R_{\text{classical}}$ and R_{excess}, respectively. The current I_1 is proportional to the average magnetic field H in the axial direction for thin magnetostrictive actuators and can be expressed as $I_1 = k_3 H$. The magnetic field H is related to the axial magnetization M via a Preisach operator as $M(\cdot) = \mathscr{W}[H(\cdot), \psi_{-1}]$, where $H(\cdot), M(\cdot) \in C[0,T]$, and ψ_{-1} is the initial memory curve [3]. The voltage V across the inductor in Fig. 1 depends on H and M via Lenz's law as in Eq. (2) below. Fig. 2 models the transduction from the magnetization to the strain in the axial direction for the actuator. In this figure, $\mathscr{W}$ is the rate-independent hysteresis operator (which in this paper will be a classical Preisach operator) that yields the axial Magnetization M from the axial magnetic field H. The quantity bM^2 is a mechanical force F that combined with an external load F_{ext}, acts as an input to the linear mechanical system yielding the strain of the magnetostrictive actuator.

It should be noted that in the original model in [1,2], the relationship between H and M appears as $M(\cdot) = \mathscr{W}[(H(\cdot) + \alpha_1 M(\cdot) + \alpha_2 y(\cdot)), \psi - 1]$, for some constants $\alpha_1, \alpha_2 \geqslant 0$. This is a more general moving model than the one studied by Brokate and Della Torre [3]. Here, we address the case $\alpha_1 = \alpha_2 = 0$. In earlier works [1,2,4,5], the

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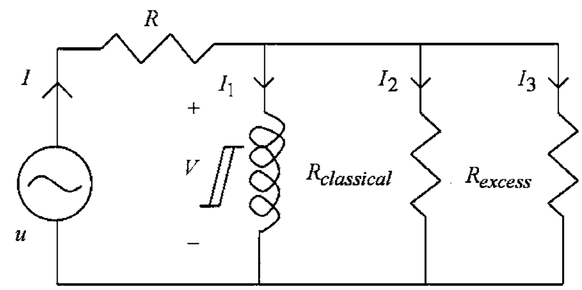

Fig. 1. A magnetostrictive actuator connected to a power supply.

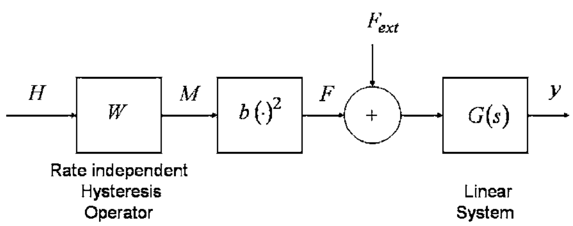

Fig. 2. Model of the transduction of the magnetic field $H(\cdot)$ to the actuator displacement $y(\cdot)$.

eddy currents losses were of the classical type with the power loss per cycle for a sinusoidal excitation of frequency f Hz given by

$$P_{\text{loss}} = P_{\text{hyst}} + P_{\text{classical}} = P_{\text{hyst}} + k_1 f \quad \text{where } k_1 > 0.$$

This model works well in the 0–50 Hz range, but for higher frequencies, it is clear that excess losses must be included. Detailed studies on soft ferromagnets [6,7] suggest that over large frequency range (from near 0 Hz to 100 KHz), the power loss consists of excess, hysteresis and classical eddy current losses. It has also been found by several researchers (see for example [8]) that laminating the ferromagnets did not reduce the excess loss vis-a-vis the classical losses. In fact for magnetostrictive actuators, the problem is accentuated because of the use of magnetic material inside the actuator casing provides a path for the flux. Based on the work by Bertotti [9], and Fiorillo and Novikov [7] on soft ferromagnets, we propose to consider the power loss per cycle to be for a sinusoidal excitation of frequency f Hz given by

$$\begin{aligned} P_{\text{loss}} &= P_{\text{hyst}} + P_{\text{classical}} + P_{\text{excess}} \\ &= P_{\text{hyst}} + k_1 f + k_2 \sqrt{f} \quad \text{where } k_1,\ k_2 > 0. \end{aligned}$$

It has been shown in [7] that this is equivalent to considering the P_{excess} as proportional to $\sqrt{|\dot{B}|}$, where $B(t)$ is the time-varying average magnetic flux density in the thin rod actuator. Since $V = K_3/\mu_0 \mathrm{d}B/\mathrm{d}t$ for some constant K_3, adding the excess losses to the model is equivalent to introducing a nonlinear resistor $R_{\text{excess}} = K_2\sqrt{|V(t)|}$, in parallel to $R_{\text{classical}}$ where $V(t)$ is the voltage shown in Fig. 1. This leads to the following time dependent coupled equations

$$K_1 HR + V\left(\frac{R}{R_{\text{classical}}} + 1\right) + \operatorname{sign}(V)(|V|)^{1/2}\frac{R}{K_2} = u, \tag{1}$$

$$K_3\left(\frac{\mathrm{d}M}{\mathrm{d}t} + \frac{\mathrm{d}H}{\mathrm{d}t}\right) = V, \tag{2}$$

$$M(\cdot) = \mathcal{W}[H(\cdot);\psi_{-1}], \tag{3}$$

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + c_1\frac{\mathrm{d}y}{\mathrm{d}t} + c_2 y = bM^2 + F_{\text{ext}}. \tag{4}$$

The constants $K_1, K_2, K_3, m > 0$ and $R, c_1, c_2, b \geqslant 0$. Eq. (1) arises from Kirchoff's voltage and current laws applied to Fig. 1. Eq. (2) is Lenz's law applied to the magnetostrictive rod. Eq. (3) relates the average Magnetization in the axial direction of the rod to the average magnetic field via a classical Preisach operator. Finally, Eq. (4) relates the displacement of the tip of the actuator y to the magnetomotive force bM^2 and the external force F_{ext} while taking into account viscous damping and elastic effects in the material.

Tan and Baras in [4,5] consider the model *without excess losses* and show the existence (and uniqueness) of solutions (H, M) in the space $C[0,T] \times C[0,T]$. However, a stronger result that (H, M) are not only continuous, *but also differentiable* (in light of Eq. (2)) needs to be shown. We show that the outline of Brokate and Sprekel's arguments for the heat equation with hysteresis [3] can be used to conclude existence and uniqueness, in spite of the nonlinear resistor R_{excess}.

2. Existence and uniqueness of solutions

Let the Preisach operator $\mathcal{W}$ be a map $\mathcal{W} : C[0,T] \to C[0,T]$ such that

H1. $\mathcal{W}$ is continuous on $C[0,T]$;
H2. $\mathcal{W}$ is piecewise increasing;
H3. The Preisach density function has a bounded integral (see assumption H4 in [3, p. 137]).

Consider Eqs. (1)–(3) in the variational form (for appropriate α, β and function f)

$$\alpha H + V + \beta \operatorname{sign}(V)\sqrt{|V|} = f, \tag{5}$$

$$\int_0^T \frac{\mathrm{d}H}{\mathrm{d}t}\phi\,\mathrm{d}t + \int_0^T \frac{\mathrm{d}M}{\mathrm{d}t}\phi\,\mathrm{d}t = \int_0^T V\phi\,\mathrm{d}t \quad \forall \phi \in L^2(0,T), \tag{6}$$

$$M(\cdot) = \mathcal{W}[H(\cdot);\psi_{-1}]. \tag{7}$$

We leave (4) in its original form because existence and uniqueness for y will follow from ODE theory once they are established for M. Consider a discretization of the time interval $[0, T]$ given by $0 = t_0 < \cdots < t_N = T$. Let H^i denote the value of $H(t)$ at these points and similar notation be used for the other functions. Eqs. (5)–(7) in discretized form become

$$\alpha H^i + V^i + \beta\,\mathrm{sign}(V^i)\sqrt{|V^i|} = f^i, \tag{8}$$

$$H^i - H^{i-1} + M^i - M^{i-1} = hV^i, \tag{9}$$

$$M^i = \mathscr{W}[H^0, \ldots, H^i; \psi_{-1}] \tag{10}$$

for $i = 1, \ldots, N$ with H^0, $\mathscr{W}[H^0; \psi_{-1}]$ initial conditions.

Lemma 2.1. *Given f^i, for each H^i in* (8) *we have a unique V^i and vice versa. Furthermore, we have $|V^i|^2 \leqslant 2\alpha^2|H^i|^2 + 2|f^i|^2$ for all $i = 1, \ldots, N$.*

Proof. Given f^i and H^i, as $h(v) = v + \beta\,\mathrm{sign}(v)\sqrt{|v|}$ is an increasing function of v, we have a unique solution V^i to the equation $h(V^i) = -\alpha H^i + f^i$. Similarly, given f^i and V^i we have a unique H^i. Next, we multiply both sides of (8) by V^i to get: $\alpha H^i V^i + |V^i|^2 + \beta|V^i|\sqrt{|V^i|} = f^i V^i$. As $\beta|V^i|\sqrt{|V^i|} \geqslant 0$, we have: $|V^i|^2 \leqslant |f^i - \alpha H^i||V^i|$. On using Cauchy–Schwartz twice on the RHS, we get: $|V^i|^2 \leqslant 2(|f^i|^2 + \alpha^2|H^i|^2)$. $\square$

Lemma 2.2. *Given f^i; $i = 1, \ldots, N$, Eqs.* (8)–(10) *with initial condition H^0 and $\mathscr{W}[H^0; \psi_{-1}]$ have a solution (V^i, H^i, M^i) for $i = 1, \ldots, N$.*

Proof. We prove this lemma by induction by proving that (8)–(9) has a solution (V^i, H^i). We do not need Eq. (10) due to the hypothesis H2. Suppose that there is a solution to the equations for $i - 1$. Eqs. (8)–(9) can be seen as the vector equation

$$S(V^i, H^i) = \begin{bmatrix} \alpha h H^i + hV^i + h\beta\,\mathrm{sign}(V^i)\sqrt{|V^i|} - hf^i \\ \alpha(H^i - H^{i-1}) + \alpha(M^i - M^{i-1}) - \alpha h V^i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

It is straightforward to see that $\lim_{\|(V^i,H^i)\| \to \infty} S(V^i, H^i) \cdot [V^i(H^i - H^{i-1})]^{\mathrm{T}} / \|(V^i, H^i)\| \to \infty$, using Hypothesis H2. As $S(V^i, H^i)$ is coercive with respect to the point $(0, H^{i-1})$, Eqs. (8)–(9) have a solution (V^i, H^i) by Proposition 1.3.1 of [3]. The existence of H^i yields the existence of M^i in Eq. (10). By using the similar argument for all i the lemma yields. $\square$

Theorem 2.1 (*Existence of solutions*). *Let the Preisach operator $\mathscr{W}$ satisfy the hypotheses* H1–H3, *f be in $L^2(0, T) \cap L^\infty(0, T)$ and $F_{\mathrm{ext}} \in L^2(0, T)$. Then the system* (1)–(4) *has a weak solution $(H, M, V, y) \in H^1(0, T) \cap L^\infty(0, T) \times H^1(0, T) \times L^2(0, T) \cap L^\infty(0, T) \times H^1(0, T)$.*

Proof. Let $h = T/N$ be the step of a discretization σ_N of the interval $[0, T]$ in N subintervals. Since $M \in H^1(0, T)$ and $F_{\mathrm{ext}} \in L^2(0, T)$ implies $y \in H^1(0, T)$ we can discuss only the system (5)–(7).

Let $H = H_h$ and $M = M_h$ be linear approximations over the discretization σ_N in $H^1(0, T)$. We denote by (H_h^i, M_h^i) the values of (H_h, M_h) at the vertices $i = 0, 1, 2, \ldots, N$ of such approximation. Also let $\bar{H}_h, \bar{V}_h$ be a constant piecewise approximation such that H_h^i, V_h^i are the constant value in the subinterval i. The approximation $(H_h, \bar{H}_h, M_h, \bar{V}_h)$ satisfies for all $\phi \in L^2(0, T)$:

$$\int_0^T \frac{\mathrm{d}H_h}{\mathrm{d}t}\phi\,\mathrm{d}t + \int_0^T \frac{\mathrm{d}M_h}{\mathrm{d}t}\phi\,\mathrm{d}t + \int_0^T \alpha\bar{H}_h\phi\,\mathrm{d}t + \beta\int_0^T \mathrm{sign}(\bar{V}_h)\sqrt{|\bar{V}_h|}\phi\,\mathrm{d}t = \int_0^T f\phi\,\mathrm{d}t, \tag{11}$$

$$M_h(\cdot) = \mathscr{W}[H_h(\cdot); \psi_{-1}]. \tag{12}$$

In order to prove the theorem, we first construct a sequence of solutions $\{(H_h^i, M_h^i, V_h^i)\}_{i=0}^N$ for different h. Then we prove that the sequence has a limit, and finally pass to the limit Eqs. (11)–(12) as h tends to zero. The construction of the sequence for different h is obtained by choosing $\phi = \phi_h$ as a standard constant piecewise approximation in $L^2(0, T)$. Then (11)–(12) yields

$$\frac{(H^i - H^{i-1})}{h} + \frac{(M^i - M^{i-1})}{h} + \alpha H^i + \beta\,\mathrm{sign}(V_i)\sqrt{|V^i|} = f^i, \tag{13}$$

$$M^i = \mathscr{W}[H^0, \ldots, H^i; \psi_{-1}], \tag{14}$$

where $f^i = \int_{(i-1)h}^{ih} f(t)\,\mathrm{d}t$ for $i = 1, \ldots, N$. The above system has a solution in agreement to Lemma 2. Therefore, we can construct a sequence of solutions for h tending to zero. Now we use compactness arguments to prove that convergent subsequences can be found. We choose $\phi_h = h\,\mathrm{d}H_h/\mathrm{d}t$ in $L^2(0, T)$, which is a piecewise constant function since H_h is linear, and obtain

$$\frac{|H^i - H^{i-1}|^2}{h} + \frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} + \alpha H^i(H^i - H^{i-1}) = f^i - \beta\,\mathrm{sign}(V^i)|V^i|^{1/2}(H^i - H^{i-1}).$$

By applying the Schwartz and the Young's inequalities and Lemma 2.1 on the RHS, we have

$$\begin{aligned} &\frac{|H^i - H^{i-1}|^2}{2h} + \frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} \\ &\quad + \alpha H^i(H^i - H^{i-1}) \leqslant \frac{h}{2}(|f^i|^2 + \beta^2|V^i|) \\ &\quad \leqslant \frac{h}{2}\left(|f^i|^2 + \frac{2\alpha T\beta^4}{2} + \frac{|V^i|^2}{4\alpha T}\right) \\ &\quad \leqslant \frac{h}{2}\left(|f^i|^2 + \beta^4\alpha T + \frac{2(\alpha^2|H^i|^2 + |f^i|^2)}{4\alpha T}\right). \end{aligned} \tag{15}$$

By induction: $\sum_{i=1}^n H^i(H^i - H^{i-1}) = \sum_{i=1}^n |H^i - H^{i-1}|^2/2 + (|H^n|^2/2 - |H^0|^2)/2$. If we sum (15) over all $i = 1, 2, \ldots, n$, where $n \leqslant N$, we have

$$\sum_{i=1}^{n} \frac{h}{2}\frac{|H^i - H^{i-1}|^2}{h^2} + \sum_{i=1}^{n} \frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} + \frac{\alpha}{2}\sum_{i=1}^{n} |H^i - H^{i-1}|^2 + \frac{\alpha}{2}(|H^n|^2 - |H^0|^2) \leqslant \frac{1}{2}\sum_{i=1}^{n} h\left(|f^i|^2 + \beta^4\alpha T + \frac{(\alpha^2|H^i|^2 + |f^i|^2)}{2\alpha T}\right). \quad (16)$$

From the theorem hypotheses f is in $L^2[0,T]$, i.e., $\sum_{i=1}^{N} h|f^i|^2 \leqslant \|f\|_{L^2} < C_1$, and therefore, taking the max over $1 \leqslant n \leqslant N$, we have

$$\sum_{i=1}^{N} \frac{h}{2}\frac{|H^i - H^{i-1}|^2}{h^2} + \sum_{i=1}^{N} \frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} + \frac{\alpha}{2}\sum_{i=1}^{N} |H^i - H^{i-1}|^2 + \frac{\alpha}{2}(\|H_h\|^2_{L^\infty} - |H^0|^2) \leqslant C_2 + \frac{\alpha}{4T}\sum_{i=1}^{N} h|H^i|^2 \quad \text{for some } C_2 > 0. \quad (17)$$

By hypothesis H2

$$\frac{1}{2}\left\|\frac{\mathrm{d}H_h}{\mathrm{d}t}\right\|_{L^2} + \frac{\alpha}{2}\sum_{i=1}^{n} |H^i - H^{i-1}|^2 + \frac{\alpha}{2}\|H_h\|^2_{L^\infty} \leqslant C_2 + \frac{\alpha}{4}\|H_h\|^2_{L^\infty}$$

$$\frac{1}{2}\left\|\frac{\mathrm{d}H_h}{\mathrm{d}t}\right\|_{L^2} + \frac{\alpha}{2}\sum_{i=1}^{n} |H^i - H^{i-1}|^2 + \frac{\alpha}{4}\|H_h\|^2_{L^\infty} \leqslant C_2,$$

and therefore the norms $\|H_h\|^2_{L^\infty}$, $\|\mathrm{d}H_h/\mathrm{d}t\|_{L^2}$ and $\sum_{i=1}^{n}|H^i - H^{i-1}|^2$ are bounded for all h. From the definition of H_h and $\bar{H}_h$ we have

$$\|H_h - \bar{H}_h\|_{L^2(0,T)} = \frac{T}{3N}\sum_{i=1}^{N} |H^i - H^{i-1}|^2, \quad (18)$$

which tends to 0 as $N \to \infty$ since $\sum_{i=1}^{N}|H^i - H^{i-1}|^2$ remains bounded. By Lemma 2.1

$$\|V_h\|^2 \leqslant (\alpha^2\|H_h\|^2 + \|f_h\|^2) \leqslant C_3 \quad \text{for some } C_3 > 0. \quad (19)$$

By the above results and by Eq. (6), we have:

$$\left|\int_0^T \frac{\mathrm{d}M_h}{\mathrm{d}t}\phi\,\mathrm{d}t\right| \leqslant C_4\|\phi\|_{L^2(0,T)} \quad \forall\phi \in L^2(0,T) \quad \text{for some } C_4 > 0. \quad (20)$$

Therefore, it is possible, from the previous sequences, to extract subsequences such that

$$\{H_h\}_j \to H \quad \text{weakly-star in } H^1(0,T) \cap L^\infty(0,T), \quad (21)$$

$$\{\bar{H}_h\}_j \to \bar{H} \quad \text{weakly-star in } L^\infty(0,T), \quad (22)$$

$$\{M_h\}_j \to M \quad \text{weakly in } H^1(0,T), \quad (23)$$

$$\{V_h\}_j \to V \quad \text{weakly-star in } L^2(0,T) \cap L^\infty(0,T). \quad (24)$$

By using these results and the fact that, from (18), $\bar{H} = H$ and $f_h \to f$ as h tends to zero we can pass to the limit (11) and obtain the desired result if the hysteresis operator equation holds in these spaces. In order to prove this, we note that the compactness and continuity of the imbedding of $H^1(0,T)$ in $C[0,T]$ (see [3, p. 17]) yields that the subsequence H_h converges also in $C[0,T]$. Let $\bar{M}_h = \mathscr{W}[H_h]$. Since $H_h \to H$, by strong continuity we have that $\bar{M} = \mathscr{W}[H]$, and the theorem follows if $M = \bar{M}$. By assumption H3 and Proposition 2.4.11 in [3], the Preisach operator $\mathscr{W}$ is Lipschitz continuous on $C[0,T]$. Therefore

$$\|M_h - \bar{M}_h\|_\infty = \|\mathscr{W}[H_h;\psi_{-1}] - \mathscr{W}[\bar{H}_h;\psi_{-1}]\|_\infty \leqslant C_5\|H_h - \bar{H}_h\|_\infty \quad \text{for some } C_5 > 0.$$

Therefore $M_h - \bar{M}_h \to 0$ strongly in $C[0,T]$.

Let $F_{\text{ext},h}$ be a linear approximation of F_{ext} over the discretization σ_N in $L^2(0,T)$. The regularity of y comes from standard theory since both M_h and $F_{\text{ext},h}$ are in $H^1(0,T)$ and therefore in $C[0,T]$. □

Next, we prove uniqueness by using the idea behind Hilpert's inequality as found in the proof of Theorem 3.3.7 in [3]. As noted in [3], Hilpert's inequality is not directly applicable when the hysteresis operator is a Preisach operator, due to its non-local memory. But the idea can still be applied by taking advantage of the definition of this operator. We need to introduce one more notation before the uniqueness result can be shown. A Preisach operator $\mathscr{W}$ is defined by an output mapping Q on the space of memory curves Ψ_0 (see [3, p. 52]) of the form

$$Q(\phi) = \int_0^\infty q(r,\phi(r))\,\mathrm{d}v(r) + w_{00}, \quad (25)$$

where v is a finite Borel measure on $\mathbb{R}_+$, $w_{00} \in \mathbb{R}$ and $q(r,s) = 2\int_0^s \omega(r,\sigma)\,\mathrm{d}\sigma$ for a given function $\omega \in L^1_{\text{loc}}(\mathbb{R}_+ \times \mathbb{R}; v \otimes \lambda)$. Corresponding to an input $H \in C[0,T]$, the memory curve $\psi \in \Psi_0$ at time $t \in [0,T]$ is given by [3]: $\psi_i(t,r) = \mathscr{F}_r[H(\cdot)\xi_{[0,t]};\psi_{-1}(r)]$, where ψ_{-1} is the "initial" memory curve at $t = 0$; $\xi(\cdot)$ is the characteristic function; $\mathscr{F}_r$ is the Play operator with parameter r.

Theorem 2.2 (*Uniqueness of solutions*). *Let the Preisach operator $\mathscr{W}$ satisfy Hypotheses* H1–H3, *and let f be in $L^2(0,T) \cap L^\infty(0,T)$ and $F_{\text{ext}} \in L^2(0,T)$. Then the solution of the system* (5)–(7) *$(H,M,V,y) \in H^1(0,T) \cap L^\infty(0,T) \times H^1(0,T) \times L^2(0,T) \cap L^\infty(0,T) \times H^1(0,T)$ is unique.*

Proof. The proof uses Hypothesis H3 in the same fashion as Theorem 3.3.7 of Brokate and Sprekels [3]. Let

(H_1, M_1, V_1, y_1) and (H_2, M_2, V_2, y_1) be two solutions of (5)–(7). Then, we have

$$\alpha(H_1 - H_2) + h(V_1) - h(V_2) = 0, \tag{26}$$

$$\int_0^T \frac{\mathrm{d}}{\mathrm{d}t}(H_1 - H_2)\phi\,\mathrm{d}t + \int_0^T \frac{\mathrm{d}}{\mathrm{d}t}(M_1 - M_2)\phi\,\mathrm{d}t + \int_0^T (V_2 - V_1)\phi\,\mathrm{d}t = 0, \quad \forall \phi \in L^2(0,T), \tag{27}$$

with $h(V) = V + \mathrm{sign}(V)\sqrt{|V|}$ a monotone increasing function. Let $\phi = H_e(H_1 - H_2)\chi_{[0,t]}$ where $H_e(\cdot)$ is the Heaviside function, and χ the characteristic function over $[0,t]$. Denote $w_i(r,t) = q(r, \mathscr{F}_r[H_i; \psi_{-1,i}(r)](t))$ for $i = 1,2$. Then we have after integration (with $(H_1 - H_2)(0) = 0$ and $\|\psi_{-1,1} - \psi_{-1,2}\|_\infty = 0$), and applying Hilpert's inequality as used in Theorem 3.3.7 of [3]:

$$(H_1 - H_2)_+(t) + \int_0^\infty (w_1(r,t) - w_2(r,t))_+\,\mathrm{d}t + \int_0^t (V_2 - V_1)H_e(H_1 - H_2)\,\mathrm{d}t \leqslant 0, \tag{28}$$

where the function $z_+ = \max\{0, z\}$. We note that $h(V)$ is strictly monotone increasing function which implies $V_2 - V_1 > 0$ if $H_1 > H_2$. Since all terms are non-negative, Eq. (28) implies $H_1 = H_2$, $w_1 = w_2$ and $V_1 = V_2$ if $H_1 > H_2$. The same result can be proved if $H_1 \leqslant H_2$. As

$$|M_1 - M_2|(t) \leqslant \int_0^\infty |w_1(r,t) - w_2(r,t)|\,\mathrm{d}t,$$

we have $M_1(\cdot) = M_2(\cdot)$. The uniqueness of y follows from standard theory for ODE's. □

3. Conclusion

In this paper, we have considered the low-dimensional model for magnetostriction [1,2,5] and added excess eddy current losses to the model. This amounts to inserting a non-linear resistor into the electrical part of the model. For input voltages in $L^2(0,T) \cap L^\infty(0,T)$ and mechanical force inputs in $L^2(0,T)$ we have also proved existence and uniqueness of solutions for the model.

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