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On the representation of hysteresis operators of Preisach type

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Abstract

Brokate and Sprekels introduced the notion of *hysteresis operators of the Preisach type* (HOPTs), that are more general than (and include as a subset) the classical Preisach operator. In this paper, we provide a mathematical framework to the important representation problem for HOPTs by presenting two results on the approximation of the output function on the set of memory curves. One of the results shows the existence of a neural network representation when the functional is continuous, and the other shows a representation based on a multiresolution analysis when the functional is Hölder continuous and bounded on compact subsets of the space of memory curves.

Keywords: Hysteresis operators of Preisach type; Representation problem; Neural networks; Multiresolution analysis

1. Introduction

The Preisach operator is a well-known and widely used hysteresis operator in the magnetics and smart structures community. An important result that makes it useful in applications is the well-known characterization theorem [1,2] which (in rough terms) states that any hysteresis operator can be represented by a Preisach operator if and only if it is rate-independent, and satisfies the congruency and wiping-out properties. It is also widely known that the hysteresis phenomenon in ferromagnetism satisfies the wiping-out property but need not satisfy the congruency property [1,3]. In the literature, there have been several generalizations within the framework of the Preisach operator that were proposed, for example, the moving model [2] and the nonlinear Priesach model [1,4]. Recently, Brokate and Sprekels [2] introduced the notion of *hysteresis operators of the Preisach type* (HOPTs), that are more general than (and include as a subset) the classical, moving and nonlinear Preisach operators. The generating functional for these operators are formed by a composition of a family of parametrized Play operators, and an output function that maps into $\mathbb{R}$. By construction the wiping-out property is satisfied, while the nature of the output function dictates whether the congruency property is satisfied or not.

Though the theory of HOPTs was well developed by Brokate and Sprekels [2], the representation of these operators needs to be studied before they can used in practical applications. There has been very few prior results on this topic, and as far as we are aware only Serpico and Visone [5] studied the problem along these lines. They make the observation that for the identification of hysteresis operators of the Preisach type, one only needs to identify the output function on the space of memory curves. This they proceed to do by discretizing the space of memory curves, and then constructing a neural network on this discretized set. Adly and Abd-El-Hafiz [6] also use a neural network on the space of discretized memory curves, though their motivation was to approximate a Preisach output function. In both these papers, the core motivation was to obtain a better match of higher-order reversals than that obtained with a classical Preisach operator.

In this paper, we provide a mathematical framework to the *representation* problem, by presenting results on the approximation of the output function on the set of memory curves. Not surprisingly, there are multiple solutions to the representation problem depending on the assumed properties of the output function. We consider the output

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function on the space of memory curves to be continuous and show that a neural network representation exists. This result provides theoretical justification to the work of Serpico and Visone [5]. The unknown coefficients that appear in this representation are nonlinearly related to the output function—a fact which hinders their identification. Furthermore, it is desirable to obtain results on the decay of the coefficients for ease of numerical implementation. This lead us to (a) impose conditions on the modulus of regularity of the output function and (b) consider it to be bounded on compact subsets of the set of memory curves. Proceeding carefully, we show that for this case, a representation arising from a multivariate multiresolution analysis exists.

2. Hysteresis operators of Preisach type

A detailed treatment on the Preisach memory curves can be found in Brokate and Sprekels [2]. The following introductory material can be found in the same source. For brevity, we adopt the same notation as Brokate and Sprekels, and refer the reader to this source for the notions of Play operator $F_r : C_{\mathrm{pm}}[0,T] \times \mathbb{R} \to C_{\mathrm{pm}}[0,T]$, where $C_{\mathrm{pm}}[0,T]$ denotes the space of piecewise monotone continuous functions. Let the "initial memory curve" as a function of r be given by $\psi_{-1}(r)$. The resulting function on $\mathbb{R}_+$ (with t fixed): $\psi_u(t,r) = F_r[u, \psi_{-1}(r)](t)$ is exactly the memory curve for a hysteresis operator of Preisach type with input u. As this function is Lipschitz continuous with Lipschitz constant 1, define the *set of admissible memory curves* for HOPTs:

$$\Psi_0 \triangleq \{\phi \mid \phi : \mathbb{R}_+ \to \mathbb{R}, |\phi(r) - \phi(\bar{r})| \leqslant |r - \bar{r}| \quad \forall r, \bar{r} \geqslant 0,$$
$$K = R_{\mathrm{supp}}(\phi) < +\infty\},$$

where $R_{\mathrm{supp}}(\phi) \triangleq \sup\{r \mid r \geqslant 0, \phi(r) \neq 0\}$. As this set is not a compact set, we will need to restrict ourselves to the case of bounded input functions in the space $C[0,T]$ or $W^{1,p}(0,T)$. Let the input function u belong to one of the sets:

$$U_0 \triangleq \{u \in C[0,T] \mid \|u\|_\infty \leqslant M_0;\ \ M_0 > 0\}, \tag{1}$$

$$U_p \triangleq \{u \in W^{1,p}(0,T) \mid \|u\|_{W^{1,p}} \leqslant M_p;\ \ M_p > 0\},$$
$$1 \leqslant p \leqslant \infty. \tag{2}$$

We will also impose the reasonable condition $u(0) = \psi_{-1}(0)$ for all the inputs under consideration, where ψ_{-1} is a given initial memory curve. Then we can show that the set of memory curves resulting from the application of these inputs to a parametrized family of Play operators is compact:

Lemma 2.1. *Let $U = U_0$ or U_p for $1 \leqslant p \leqslant \infty$, and furthermore, suppose that all $u \in U$ satisfy $u(0) = \psi_{-1}(0)$. For each $\psi_{-1} \in \Psi_0$, the set: $\Phi_U = \bigcup_{t\in[0,T],\, u\in U} \psi_u(t,\cdot) \subset \Psi_0$, is a compact subset of $C([0,\bar{M}])$, where $\bar{M} = \max\{|\psi_{-1}(0)| + M, R_{\mathrm{supp}}(\psi_{-1})\}$ and $M = M_0$ or $M = M_p$ for $1 \leqslant p < \infty$ or $M = M_\infty T$ for $p = \infty$.*

Proof. For the case $U = U_0$, by Lemma 2.4.7 in Ref. [2], the support of the memory curves is the set $[0, R_{\mathrm{supp}}(\psi(T))]$ where $R_{\mathrm{supp}}(\psi(T)) = \max\{M_0, R_{\mathrm{supp}}(\psi_{-1})\}$. Obviously, $R_{\mathrm{supp}}(\psi(T)) \leqslant \bar{M}$ given in the statement. For the case $U = U_p$, we use the fact that a function $v \in W^{1,p}(0,T)$ if and only if $v(t) = v(0) + \int_0^t w(t)\,\mathrm{d}t$ for some $w(\cdot) \in L^p(0,T)$. It is clear that $v'(t) = w(t)$ a.e on $(0,T)$. So for $1 \leqslant p < \infty$: $|v(t)| \leqslant |v(0)| + \int_0^t |v'(s)|\,\mathrm{d}s \leqslant |v(0)| + [\int_0^t |v'(s)|^p\,\mathrm{d}s]^{1/p} \leqslant |\psi_{-1}(0)| + M_p$, and for $p = \infty$: $|v(t)| \leqslant |v(0)| + \int_0^t |v'(s)|\,\mathrm{d}s \leqslant |v(0)| + \|v'\|_\infty \int_0^t 1\,\mathrm{d}s \leqslant |\psi_{-1}(0)| + M_\infty t$.

Therefore $\sup_{0\leqslant t\leqslant T} |v(t)| \leqslant |\psi_{-1}(0)| + M$, and $M = M_0$ or $M = M_p$ for $1 \leqslant p < \infty$ or $M = M_\infty T$ for $p = \infty$. Again by Lemma 2.4.7 in Ref. [2], the support of the memory curves is the set $[0, R_{\mathrm{supp}}(\psi(T))]$ where $R_{\mathrm{supp}}(\psi(T)) = \max\{|\psi_{-1}(0)| + M, R_{\mathrm{supp}}(\psi_{-1})\}$. When $U = U_0$ or U_p for $1 \leqslant p \leqslant \infty$, the set Φ_U is a bounded, equicontinuous subset of $C[0,\bar{M}]$. Hence it is compact by the Arzela–Ascoli theorem. □

A main point of Brokate and Sprekels' HOPT is that the hysteresis operator is determined by an output function on the space of memory curves Ψ_0 [2, p. 52]. Thus the representation problem for the operator reduces to the representation problem for the output function.

3. Approximation of nonlinear continuous functionals

The key ideas in the approximation of a nonlinear continuous functional defined on Ψ_0 are: (a) Restrict the set of inputs so that the corresponding subset of memory curves Φ_U is compact. Approximate the functional on the subset Φ_U according to the steps below. (b) Project the memory curves to a finite D-dimensional subspace of $C([0,\bar{M}])$. This process is achieved by maps: $\Pi_D : \Phi_U \to C_U \subset \mathbb{R}^D$, and $L_D : \mathbb{R}^D \to \Phi_U^D \subset \Phi_U$; the first map is simply a "sampling" step, while the second step is an interpolation on the sampled values. (c) Approximate the continuous function $Q : \Phi_U^D \to \mathbb{R}$ by a neural network or through multiresolution analysis. Notice that: $Q_D = Q \circ L_D$ can be thought of directly acting on the compact subset $C_U \subset \mathbb{R}^D$ and hence $Q_D : C_U \to \mathbb{R}$. This is similar to the approach of Chen–Chen [7]. However, there is also another approach to the projection and interpolation step where one can directly project into a finite dimensional subspace of polynomials [8]. This approach is not appropriate for our problem, for the following reason. The function Q is defined on the space Ψ_0 which consists of curves of Lipschitz constant 1. The curve after projection into a finite dimensional subspace of polynomials need not belong to Ψ_0 and hence approximating Q over this set is meaningless. Due to this subtlety, the interpolation step in (b) above needs more attention.

Suppose, $0 \leqslant r_1 \leqslant \cdots \leqslant r_D$ be a discretization of $[0,\bar{M}]$. Suppose that the projection Π_D is defined by $\Pi_D : \phi \mapsto (\phi(r_1), \ldots, \phi(r_D))$. An interpolation between these points can be defined in several ways to yield a curve in Φ_U. For $r \in [r_k, r_{k+1}]$, we define the interpolation between points $(r_k, \phi(r_k))$ and $(r_{k+1}, \phi(r_{k+1}))$ to be the curve

$$\phi_+(r) = \max\{\psi(r) \mid \psi \in \Psi_0;\ \psi(r_k) = \phi(r_k);\ \psi(r_{k+1}) = \phi(r_{k+1})\}. \tag{3}$$

Other possibilities include

$$\phi_-(r) = \min\{\psi(r) \mid \psi \in \Psi_0;\ \psi(r_k) = \phi(r_k);\ \psi(r_{k+1}) = \phi(r_{k+1})\}. \tag{4}$$

In Fig. 1, a given memory curve $\phi(r)$ is shown as a solid line, and the curves $\phi_+(r)$, $\phi_-(r)$ are shown by dashed lines. It is clear that the given memory curve $\phi(r)$ is always bounded by $\phi_+(r)$ and $\phi_-(r)$. It is also clear by the definition of the sets U_0 and U_p where $1 \leqslant p \leqslant \infty$ that the curves $\phi_\pm(\cdot) \in \Phi_U$ if $\phi(\cdot) \in \Phi_U$. The maximum value of the curve $\phi_+(r)$ between r_k and r_{k+1} can be easily computed to be $\max_{r_k \leqslant r \leqslant r_{k+1}} \phi_+(r) = (\phi(r_k) + \phi(r_{k+1}))/2 + (r_{k+1} - r_k)/2$. The minimum value of the curve $\phi_-(r)$ between r_k and r_{k+1} can also be computed to be $\min_{r_k \leqslant r \leqslant r_{k+1}} \phi_-(r) = (\phi(r_k) + \phi(r_{k+1}))/2 + (r_k - r_{k+1})/2$, while the points at which the minimum and maximum are attained are, respectively (for all cases $\phi(r_k) \gtreqless \phi(r_{k+1})$): $x_1 = (\phi(r_k) - \phi(r_{k+1}))/2 + (r_k + r_{k+1})/2$; $x_2 = (\phi(r_{k+1}) - \phi(r_k))/2 + (r_k + r_{k+1})/2$. With this information, we compute (for all cases $\phi(r_k) \gtreqless \phi(r_{k+1})$)

$$\begin{aligned}\sup_{r_k \leqslant r \leqslant r_{k+1}} |\phi_+(r) - \phi_-(r)| &= \phi_+(x_2) - \min_{r_k \leqslant r \leqslant r_{k+1}} \phi_-(r) \\ &= (r_{k+1} - r_k) - |\phi(r_k) - \phi(r_{k+1})|. \end{aligned} \tag{5}$$

Hence, the interpolation $\phi_+(r)$ and the given memory curve $\phi(r)$ satisfy

$$\begin{aligned}\sup_{r_k \leqslant r \leqslant r_{k+1}} |\phi_+(r) - \phi(r)| &\leqslant \sup_{r_k \leqslant r \leqslant r_{k+1}} |\phi_+(r) - \phi_-(r)| \\ &= (r_{k+1} - r_k) - |\phi(r_k) - \phi(r_{k+1})|.\end{aligned}$$

$$\begin{aligned}\|\phi_+ - \phi\|_{L^\infty} &= \max_{k=1,\ldots,D-1} (r_{k+1} - r_k) - |\phi(r_k) - \phi(r_{k+1})| \\ &\leqslant \max_{k=1,\ldots,D-1} (r_{k+1} - r_k).\end{aligned} \tag{6}$$

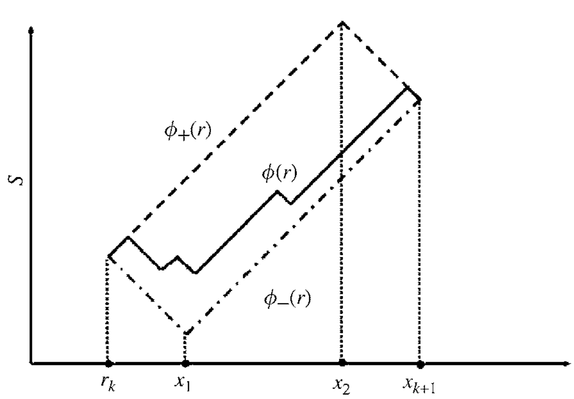

Fig. 1. Interpolation between two sampled values $\phi(r_1)$ and $\phi(r_2)$.

This result tells us that for a given $\phi \in \Psi_0$, if the net $r_1 \leqslant \cdots \leqslant r_D$ is dense in the $R_{\text{supp}}(\phi)$ in the limit then $\lim_{D\to\infty} \|\phi_+ - \phi\|_{L^\infty} = 0$. Another fact that can be easily verified using the same technique as above is $\|\phi_+ - \psi_+\|_{L^\infty} = \|\Pi_D(\phi) - \Pi_D(\psi)\|_\infty$.

3.1. Approximation with neural networks

We next study the approximation of the continuous functional Q by neural networks. In Serpico–Visone [5] and Adly–Abd-Al-Hafiz [6], a neural network was constructed using sigmoidal functions to approximate Q_D. However, more general constructions are possible as shown by Chen–Chen [9,7].

Definition 3.1 (*TW function*) (*Chen and Chen [7]*). If $g : \mathbb{R} \to \mathbb{R}$ satisfies that all linear combinations: $\sum_{i=1}^N c_i g(\lambda_i x + \theta_i)$, $\lambda_i \in \mathbb{R}$, $\theta_i \in \mathbb{R}$, $i = 1, \ldots, N$, are dense in every $C[a,b]$, then g is called a Tauber–Wiener (TW) function.

Sigmoidal functions that are widely used in neural network literature are TW functions [7].

Theorem 3.1. *Suppose Q is a continuous functional on Ψ_0. Let $U = U_0$ or U_p for $1 \leqslant p \leqslant \infty$, and suppose that all $u \in U$ satisfy $u(0) = \psi_{-1}(0)$. Furthermore, suppose that g is a TW function. Then for any $\varepsilon > 0$, there are D points $r_1, \ldots, r_D \in [0, \bar{M}]$; a positive integer N; and real constants c_i, θ_i, ξ_{ij}, $i = 1, \ldots, N$, $j = 1, \ldots, D$* such that

$$\left| Q(\phi) - \sum_{i=1}^N c_i g\left(\sum_{j=1}^D \xi_{ij} \phi(r_j) + \theta_i \right) \right| < \varepsilon \quad \forall \phi \in \Phi_U. \tag{7}$$

Proof. Φ_U is a compact subset of $C([0, \bar{M}])$ when $U = U_0$ or U_p by Lemma 2.1. So the theorem follows from Theorem 4 of Chen and Chen [7]. □

It is interesting to note here that the Tietze Extension theorem is used in the proof of Theorem 4 in Ref. [7], while we will need the Whitney–McShane Extension Lemma [10] in the next section.

3.2. Approximation with multiresolution analysis

Theorem 3.1 is an existence theorem and does not yield any conditions on the coefficients c_i, θ_i, ξ_{ij} as $N \to \infty$. This is because we have not imposed any conditions on Q other than continuity. Using the framework of multiresolution analysis we can hope to characterize the coefficients if we imposed some restriction on the modulus of continuity of Q—that is defined as [2]

$$\omega_\infty(Q; \delta) \triangleq \sup\{|Q(\phi) - Q(\psi)| \mid \phi, \psi \in \Psi_0;\ \|\phi - \psi\|_{L^\infty} \leqslant \delta\}. \tag{8}$$

The functional Q is said to be α-Hölder continuous if

$$|Q(\phi) - Q(\psi)| \leqslant C\|\phi - \psi\|^\alpha_{L^\infty}; \quad \phi, \psi \in \Psi_0 \tag{9}$$

for some $C>0$ and $0<\alpha<1$. The smallest number C for which Eq. (9) is true is called the Hölder semi-norm $[Q]_\alpha$ of Q. It is easy to see that Q is α-Hölder continuous with if and only if $\omega_\infty(Q;\delta)\leqslant C\delta^\alpha$. One possible restriction that one might impose on Q is for it to be α-Hölder continuous. However, as we will be approximating of Q_D, it is the Hölder continuity of Q_D that is required. The modulus of continuity of Q_D can be defined to be

$$\omega_\infty(Q_D;\delta)\triangleq \sup\{|Q_D(\Pi_D(\phi))-Q_D(\Pi_D(\psi))| \\ |\phi,\psi\in\Psi_0;\ \|\Pi_D(\phi)-\Pi_D(\psi)\|_\infty\leqslant\delta\}. \quad (10)$$

The first result is straight-forward. If Q is α-Hölder continuous and $\phi\in\Psi_0$, then

$$\lim_{D\to\infty}|Q(\phi)-Q_D(\Pi_D(\psi))| = \lim_{D\to\infty}|Q(\phi)-Q(\phi_+)| \\ \leqslant \lim_{D\to\infty} C\|\phi-\phi_+\|^\alpha_{L^\infty}=0.$$

We now suppose that a discretization $r_1\leqslant\cdots\leqslant r_D$ has been selected. Let $\varphi\in\Psi_0$, and let $\phi=\varphi_+$ and $\psi=\varphi_-$, where the interpolated curves are as defined in Eqs. (3) and (4). Then from Eq. (6) we see that $\|\phi-\psi\|_{L^\infty}=\max_{k=1,\dots,D-1}(r_{k+1}-r_k)$. In the finite dimensional space R^D we have $\|\Pi_D(\phi)-\Pi_D(\psi)\|_\infty=0$. It is clear that we can change the values of ϕ by at most $(r_{k+1}-r_k)$ in the interval (r_k,r_{k+1}) and still have $\|\Pi_D(\phi)-\Pi_D(\psi)\|_\infty=0$. On the other hand, we can have curves $\phi,\psi\in\Psi_0$, that differ by a constant, yielding $\|\Pi_D(\phi)-\Pi_D(\psi)\|_\infty=\|\phi-\psi\|_{L^\infty}$. Hence

$$\omega_\infty(Q_D,\delta)=\sup_{\|\Pi_D(\phi)-\Pi_D(\psi)\|_\infty\leqslant\delta}|Q_D(\Pi_D(\phi))-Q_D(\Pi_D(\psi))| \\ \leqslant \sup_{\|\phi-\psi\|_{L^\infty}\leqslant\delta}|Q(\phi)-Q(\psi)| \\ =\omega_\infty(Q,\delta).$$

Hence any restriction on the modulus of continuity of Q passes on to Q_D. Furthermore, the Hölder semi-norm $[Q_D]_\alpha\leqslant[Q]_\alpha$.

We now turn our attention to multiresolution analysis. A multiresolution analysis of $L^2(\mathbb{R}^D)$ consists of closed subspaces $(V_j)_{j\in\mathbb{Z}}$ of $L(\mathbb{R}^D)$ having the following properties [11,12]:

1. $\cdots\subset V_1\subset V_0\subset V_{-1}\subset\cdots$.
2. $\bigcup_{j\in\mathbb{Z}}V_j$ is dense in $L^2(\mathbb{R}^D)$ and $\bigcap_{j\in\mathbb{Z}}V_j=\{0\}$.
3. $f\in V_j\Leftrightarrow f(2^{-j}\cdot)\in V_0$.
4. $f\in V_0 =;\Leftrightarrow f(\cdot-\gamma)\in V_0$ for all $\gamma\in\mathbb{Z}^D$.
5. There exists a function called the scaling function $\phi\in V_0$, such that the system $\{\phi(\cdot-\gamma)\}_{\gamma\in\mathbb{Z}^D}$ is an orthonormal basis in V_0.

Even though the D-dimensional Haar wavelet is a multiresolution analysis on $L^2(\mathbb{R}^D)$, it is not an appropriate tool to approximate α-Hölder continuous functions, as we need wavelets with more regularity. For our purposes, we only need the following definition (modified from Ref. [11]):

Definition 3.2. A multiresolution analysis on $\mathbb{R}^D$ is called regular if the scaling function ϕ is of class C^1, and satisfies: $|(\partial/\partial x)\phi(x)|\leqslant C_k/(1+|x|)^k$, for each $k=0,1,2,\dots$ and some constants C_k.

It is known that there exist regular multiresolution analysis for $L^2(\mathbb{R}^D)$ [11–13], with an orthonormal wavelet set $\{\psi_{i,j,k}(x)=2^{-Dj/2}\psi_i(2^{-j}x-k),\ i=1,2,\dots,2^D-1,\ j\in\mathbb{Z},\ k\in\mathbb{Z}^D.\}$. If ϕ_0 is the scaling function, then we denote its translates in V_0 by $\phi_{0,k}=\phi_0(x-k)$.

Let us now consider the approximation of Q_D using multiresolution analysis. As Q_D is only defined on a compact subset C_U of $\mathbb{R}^D$, we extend it to a α-Hölder continuous function $Q_D^{\rm ext}$ defined on all of $\mathbb{R}^D$; agreeing with Q_D on C_U; and having the same Hölder semi-norm as Q_D. This is done by an easy extension of Whitney–McShane Lemma [10]. For instance, the function $Q_D^{\rm ext}(x)=\inf_{a\in C_U}Q_D(a)+[Q_D]_\alpha\|x-a\|^\alpha$ where $x\in\mathbb{R}^D$, satisfies the requirements.

The projection of $Q_D^{\rm ext}(x)$ where $x\in\mathbb{R}^D$ onto V_0 is given by $\beta_{0,k}=\int Q_D^{\rm ext}(x)\bar\phi_{0,k}\,{\rm d}x,\ k\in\mathbb{Z}^D$. Here $\bar\phi_{0,k}$ denotes the complex conjugate of $\phi_{0,k}$. The projection of $Q_D^{\rm ext}(x)$ onto the wavelet subspaces is given by $\alpha_{i,j,k}=2^{-Dj/2}\int Q_D^{\rm ext}(x)\bar\psi_{i,j,k}\,{\rm d}x,\ i=1,2,\dots,2^D-1,\ j\in\mathbb{Z},\ k\in\mathbb{Z}^D$. Then we have the following theorem [12]:

Theorem 3.2. *Suppose that $Q:\Psi_0\to\mathbb{R}$ is s-Hölder continuous for some $0<s<1$ and Q is bounded on compact subsets of Ψ_0. Let $U=U_0$ or U_p for $1\leqslant p\leqslant\infty$ and $u(0)=\psi_{-1}(0)$ for all $u\in U$. Let the support of the memory curves in Φ_U be contained in $[0,\bar M]$ and let $0=r_1\leqslant\cdots\leqslant r_D=\bar M$be a discretization of $[0,\bar M]$. Then $Q_D^{\rm ext}$ is s-Hölder continuous $(0<s<1)$ if and only if in a regular multiresolution approximation the coefficients satisfy $|\beta_{0,k}|\leqslant C_0;\ k\in\mathbb{Z}^D$ and $|\alpha_{i,-j,k}|\leqslant C_1 2^{-j(s+\frac{Dj}{2})},\ i=1,2,\dots,2^D-1,\ j\geqslant0,\ k\in\mathbb{Z}^D$, for some $C_0,C_1>0$.*

Proof. The assumption that Q is bounded on compact subsets of $\Psi_0$implies that $Q_D^{\rm ext}\subset L^1_{\rm loc}(\mathbb{R}^D)$. This and the assumption on the α-Hölder continuity of Q yield the theorem by Theorem 5, Chapter 6 of Meyer [12]. □

Theorem 3.3. *Suppose Q and the input signals satisfy the conditions of Theorem 3.2. Let $R_{\rm supp}(\Phi_U)\leqslant\bar M$. Then given an $\varepsilon>0$, there exists $0=r_1\leqslant\cdots\leqslant r_D$ of $[0,\bar M]$ such that*

$$\Bigg|Q(\phi)-\sum_{k\in\mathbb{Z}}\beta_{0,k}\,\phi_{0,k}(\Pi_D(\phi)) \\ +\sum_{j\geqslant0}\sum_{i=1}^{2^D-1}\sum_{k\in\mathbb{Z}^D}\alpha_{i,-j,k}\,\psi_{i,-j,k}(\Pi_D(\phi))\Bigg|<\varepsilon;\quad \phi\in\Psi_0, \quad (11)$$

where the coefficients $\beta_{0,k}$ and $\alpha_{i,-j,k}$ satisfy the conditions of Theorem 3.2.

Proof. The proof follows from the fact that as the net $0=r_1\leqslant\cdots\leqslant r_D$ becomes dense in $[0,\bar M]$ for $D\to\infty$, we have

$\lim_{D\to\infty}|Q(\phi) - Q(\phi_+)| = \lim_{D\to\infty}|Q(\phi) - Q_D(\Pi_D(\phi))| = 0$, and Theorem 3.2. □

4. Conclusion

In this paper, we propose two approaches to the *representation* problem for hysteresis operators of Preisach type (HOPTs). One of the results shows the existence of a neural network representation when the functional is continuous, and the other shows a representation based on a multiresolution analysis when the functional is Hölder continuous and locally bounded the set of memory curves.

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