Complex Analysis – Homework 3

1. Find and classify the isolated singularities of the function

$$f(z) = \frac{e^{1/z} - e}{z - 1} + \frac{1}{\sin z}.$$

2. Let $\gamma(t) = 3e^{it}$, $0 \leq t \leq 2\pi$. Compute the integrals

$$\int_\gamma \tan z dz,$$
$$\int_\gamma \frac{1}{\sin z \cos z} dz$$

3. Use Rouché's theorem to find the number of solutions of $z^5 + 3z^2 + 1 = 0$ in the set $A = \{z \,|\, 1 \leq |z| \leq 2\}$.