Complex Analysis – Homework 1

1. Find the radius of convergence of $\sin z$ and $\cos z$.

2. Find the radius of convergence for the following power series

$$\sum_{n=0}^{\infty} n^2 z^n, \quad \sum_{n=0}^{\infty} \frac{2^n z^{2n}}{n^2+n}.$$

3. Find the radius of convergence of the power series that is the Taylor series of

$$f(z) = \frac{2z+3}{4z+5}$$

at the origin.

4. Find the image of the sets

$$\{z \,|\, \mathrm{Re} z < 0, |\mathrm{Im} z| < \pi\}$$
$$\{z \,|\, |\mathrm{Im} z| < \pi/2\}$$

under the exponential map.

5. Discuss the mapping properties of $\cos z$, $\sin z$, z^n, $n \geq 2$.

6. Find the Möbius transformations that map the unit disk onto itself.

7. Find a holomorphic map that maps $\{z \,|\, \mathrm{Re} z > \mathrm{Im} z\}$ to $\{z \,|\, |z-1| < 2\}$.