

Homework 6

1. Prove that for every real number x , $e^x \geq x + 1$.
2. Find all positive real solutions to the equation $2^x = x^2$.
3. Show that for every positive numbers a, b , and positive integer n , one has $a^n + (n - 1)b^n \geq nab^{n-1}$.
4. Compute the following limits using l'Hospital's theorem:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x^2}$$

$$\lim_{x \rightarrow 0+} x \ln \sin x$$

$$\lim_{x \rightarrow 0+} x^x.$$