

Homework 4

1. Using an $\epsilon - \delta$ argument, prove that $f(x) = x^3$ is continuous at every point.
2. What should $f(2)$ be so that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{x-2}{\sqrt{x}-\sqrt{2}}$ for $x \neq 2$ is continuous everywhere?
3. Show that the function $f(x) = \sqrt[3]{x}$ is continuous everywhere.
4. Prove that $f(x) = |x|$ is continuous at 0.
5. Show that the polynomial $f(x) = x^4 + 7x^3 - 9$ has at least two real roots.
6. Find the number a such that the function

$$f(x) = \begin{cases} x^2 + 5a & \text{if } x < 2 \\ ax + 7 & \text{if } x \geq 2 \end{cases}$$

is continuous.