Introduction to Topology – Homework 2

1. Show that if one space is a deformation retract of another, then their fundamental groups are isomorphic.

2. The Klein bottle is obtained as a quotient space of $[0,1] \times [0,1]$ by the equivalence relations $(s,0) \equiv (s,1)$ and $(0,t) \equiv (1,1-t)$, for all $s,t \in [0,1]$. Compute the fundamental group of the Klein bottle.

3. What is the fundamental group of $S^1 \bigvee S^2$?

4. Let X be the space obtained by gluing $\overline{B^2}$ to S^1 by the map on the boundary $z \to z^n$. What is the fundamental group of X?

5. Find a space whose fundamental group is $\mathbb{Z}_3 \times \mathbb{Z}_5$.