Homework 5

1. Show that the function

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = 2x^3 + 3x$$

   is continuous everywhere.

2. Let f be defined for all $x \in \mathbb{R}$, $x \neq 2$ by $f(x) = (x^2 + x - 6)/(x - 2)$. Can f be defined at $x = 2$ so that f is continuous at this point?

3. Let f be defined for all $x \in \mathbb{R}$, $x \neq 0$ by $f(x) = x \cot x$. Can f be defined at $x = 0$ so that f is continuous at this point?

4. Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for each $x \in I$. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all $x \in I$.