Homework 3

1. Let $x_1 = 5$ and
$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right) \text{ for } n \geq 1.$$
Prove that (x_n) is a convergent sequence and find its limit.

2. Let $x_1 = 1$, and $x_{n+1} = \sqrt{2 + x_n}$, $n \geq 1$. Show that (x_n) converges and find its limit.

3. Let (y_n) be defined by $y_1 = 1$ and $y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Show that (y_n) converges and find its limit.

4. Establish the convergence or divergence of the sequence
$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}, \quad n \geq 1.$$