Math 5362 - Algebraic Number Theory
Homework 2

Due in Class - Thursday 5 March 2020

1. For p an odd prime, prove that the discriminant of the cyclotomic field $\mathbb{Q}(\zeta_p)$ equals $(-1)^{\frac{p-1}{2}}p^{p-2}$.

2. Let $K = \mathbb{Q}(\sqrt[3]{2})$. Let $\alpha = a + b\sqrt[3]{2} + c(\sqrt[3]{2})^2 \in K$ be a general element. Compute

(a) $D(\sqrt[3]{2})$;

(b) $N_K(\alpha)$; and

(c) $Tr_K(\alpha)$.

3. Let $\theta = \sqrt[3]{12}$. Show that $\{1, \theta, \theta^2\}$ is not an integral basis for $K = \mathbb{Q}(\theta)$.

4. Determine the minimal polynomial of $2^{\frac{1}{3}} + \omega$ over $\mathbb{Q}(2^{\frac{1}{3}})$, where ω is a primitive cube root of unity.

5. Let $I = < 7, 3 + \sqrt{-5} >$ and $J = < 7, 3 - \sqrt{-5} >$ be ideals in $\mathbb{Z}[\sqrt{-5}]$.

(a) Calculate IJ and I^2; and

(b) Find a fractional ideal M such that $IM = \mathbb{Z}[\sqrt{-5}]$.