Math 5362 - Algebraic Number Theory
Homework 1

Due in Class - Thursday 13 February 2020

1. Prove that

$$\frac{10^{\frac{2}{3}} - 1}{\sqrt{-3}}$$

is an algebraic integer.

2. Determine for which integers m is

$$\alpha = \frac{\sqrt{m} + 1}{\sqrt{2}}$$

an algebraic integer.

3. Express the algebraic number

$$\left(\frac{1+\sqrt{2}}{9}\right)^{\frac{1}{3}} + \left(\frac{1-\sqrt{2}}{9}\right)^{\frac{1}{3}}$$

as a quotient $\frac{\alpha}{m}$, where $m \in \mathbb{Z}$ and α is an algebraic integer .

4. Determine the minimal polynomial of $\frac{1+i}{\sqrt{2}}$ over

(a) $\mathbb{Q}$;

(b) $\mathbb{Q}(i)$; and

(c) $\mathbb{Q}(\sqrt{2})$.

5. Determine $\alpha \in \mathbb{C}$ such that $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) = \mathbb{Q}(\alpha)$ and prove that $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) : \mathbb{Q}] = 8$.

6. Let $K = \mathbb{Q}(\theta)$ where $\theta^3 + 11\theta - 4 = 0$. Prove that $(\theta^2 - \theta)/2 \in \mathcal{O}_K$.

7. Let $K = \mathbb{Q}(\theta)$ where $\theta^3 - 4\theta + 2 = 0$. Let $\alpha = \theta + \theta^2$. Calculate $D(\alpha)$.