Math 4363 - Combinatorics
Homework 7
Due - Sunday 19 April 2020 11:59pm

1. Let $f(n)$ denote the Fibonacci sequence. By evaluating each of the following expressions for small values of n, conjecture a general formula and then prove it using induction and the Fibonacci recurrence.

   (a) $f(1) + f(3) + f(5) + \cdots + f(2n-1)$

   (b) $f(0) + f(2) + f(4) + \cdots + f(2n)$

   (c) $f(0) - f(1) + f(2) - \cdots + (-1)^n f(n)$

   (d) $f(0)^2 + f(1)^2 + f(2)^2 + \cdots + f(n)^2$

2. Determine the generating function for the sequence $h(n)$ of the number of ways to choose n pieces of fruit from apples, bananas, pears and oranges such that the number of

   - apples is even;
   - bananas is a multiple of 3;
   - oranges is at most 2; and
   - pears is at most 1.

   Then find a formula for $h(n)$ from the generating function.