Math 4363 - Combinatorics
Homework 5
Due in Class - Thursday 5 March 2020

1. What is the coefficient of x^5y^{13} in the expansion of $(3x-2y)^{18}$?

2. Let n be a positive integer. Prove that

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}^2=\begin{cases}0 & \text{if } n \text{ is odd}\\ (-1)^m\binom{2m}{m} & \text{if } n=2m.\end{cases}$$

Hint: For $n=2m$, consider the coefficient of x^n in $(1-x^2)^n=(1+x)^n(1-x)^n$.

3. Find a single binomial coefficient equal to

$$\binom{n}{k}+3\binom{n}{k-1}+3\binom{n}{k-2}+\binom{n}{k-3}.$$

4. Prove that, for all real numbers r and all integers m and k,

$$\binom{r}{m}\binom{m}{k}=\binom{r}{k}\binom{r-k}{m-k}.$$

5. What is the coefficient of $x_1^4\,x_2^3\,x_3^2\,x_4$ in the expansion of $(x_1+x_2+x_3+x_4)^{10}$?