Math 4363 - Combinatorics
Homework 4
Due in Class - Thursday 27 February 2020

1. Use the pigeonhole principle to show that if $n+1$ integers are chosen from the set $\{1,2,\cdots,2n\}$, then there are always two of them which differ by 1.

2. Show that for any 52 integers, two of them will have a sum or difference which is divisible by 100.

3. A child watches TV for at least one hour each day for seven weeks, but never more than 11 hours in any one week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV. (Assume that the child watches TV for a whole number of hours each day.)

4. A bag contains 100 apples, 100 bananas, 100 pears and 100 oranges. If I pick one piece of fruit out of the bag every minute, how long will it be before I am guaranteed to have at least a dozen pieces of fruit of the same kind?

5. There are 100 people at a party. Each person has an even number (possible zero) of acquaintances at the party. Prove that there are at least three people at the party with the same number of acquaintances.