

---

**Math 4362 - Number Theory**  
**Homework 5**  
**Due in Class - Thursday October 17, 2019**

---

1. Calculate
  - (a)  $\phi(5040)$ ; and
  - (b)  $\phi(3456)$ .
  
2. Prove that
  - (a) If  $d \mid n$ , then  $\phi(d) \mid \phi(n)$ ;
  - (b) If  $n$  is odd, then  $\phi(2n) = \phi(n)$ ; and
  - (c) If  $n$  is even, then  $\phi(2n) = 2\phi(n)$ .
  
3. Find all solutions of  $\phi(n) = 24$ .
  
4. For positive integers  $m$  and  $n$  prove that
  - (a)  $\phi(m)\phi(n) = \phi(mn)\phi(d)/d$ , where  $d = \gcd(m, n)$
  - (b)  $\phi(m)\phi(n) = \phi(\gcd(m, n))\phi(\text{lcm}(m, n))$ .
  
5. Use Euler's Theorem to
  - (a) evaluate  $2^{100000}$  modulo 77; and
  - (b) find the units digit of  $3^{100}$ .