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**Math 4362 - Number Theory**  
**Homework 3**  
**Due in Class - Thursday 19 September 2018**

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1. Let  $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$  be the prime factorization of some positive integer  $n > 1$ . Prove that  $n$  is a square if and only if  $a_1, a_2, \dots, a_r$  are all even.
2. Determine all solutions in the integers of the following Diophantine equations:
  - (a)  $24x + 138y = 18$
  - (b)  $14x + 35y = 93$
  - (c)  $54x + 21y = 906$
  - (d)  $158x - 57y = 7$
3. A man has \$4.55 in change composed entirely of dimes and quarters. What are the maximum and minimum number of coins that he can have? Is it possible for the number of dimes to equal the number of quarters?
4. Find the remainder when  $2^{50}$  is divided by 7.
5. Solve the following linear congruences:
  - (a)  $5x \equiv 2 \pmod{26}$
  - (b)  $36x \equiv 8 \pmod{96}$
  - (c)  $140x \equiv 133 \pmod{301}$