
Math 4362 - Number Theory

Homework 1

Due in Class - Thursday 5 September 2019

1. Use mathematical induction to prove that

(a)

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

(b)

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

2. Use the binomial theorem to show that

(a)

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

(b)

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0.$$

3. Show that any integer of the form $6t + 5$, for some integer t , is also of the form $3s + 2$, for some integer s , but that the converse is false.

4. Use the Division Algorithm to establish that the fourth power of any integer is of the form $5k$ or $5k + 1$, for some integer k .

5. Prove or disprove: if $a \mid (b + c)$ then $a \mid b$ or $a \mid c$.

6. Given integers, a, b, c, d show that

(a) if $a \mid b$ then $a \mid bc$.

(b) if $a \mid b$ and $c \mid d$, then $ac \mid bd$.