Math 4362 - Number Theory
Homework 6
Due in Class - Thursday October 25, 2018

1. Find the order of all appropriate positive integers modulo 14, and identify the primitive roots, if any.

2. Prove that

   (a) if a has order hk modulo n, then a^h has order k modulo n.

   (b) if a has order $2k$ modulo an odd prime p, then $a^k \equiv -1 \pmod{p}$.

3. Let r be a primitive root of the positive integer n. Prove that r^k is a primitive root of n if and only if $\gcd(k, \phi(n)) = 1$.

4. Determine all primitive roots of $p = 19$.

5. Given that 3 is a primitive root of 43, find the following:

   (a) all positive integers less than 43 that have order 6 modulo 43;

   (b) all positive integers less than 43 that have order 21 modulo 43; and

   (c) all other primitive roots of 43.

6. Let r be a primitive root of the odd prime p. Prove the following:

   (a) if $p \equiv 1 \pmod 4$, then $-r$ is also a primitive root of p.

   (b) if $p \equiv 3 \pmod 4$, then $-r$ has order $\frac{p-1}{2}$ modulo p.