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## Math 4362 - Number Theory

### Homework 7

Due in Class - Monday November 24, 2014

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1. (a) Find the order of all appropriate positive integers modulo 5.  
(b) Using your results from part (a), find a primitive root of 25.  
(c) How many primitive roots of 25 are there?  
(d) Using your results from part (b) find all primitive roots of 25.

2. Prove that 3 is a primitive root of all integers of the form  $7^k$ .

3. Assume that  $r$  is a primitive root of an odd prime  $p$  and that

$$(r + tp)^{p-1} \not\equiv 1 \pmod{p^2},$$

for some  $t \in \mathbb{Z}^+$ . Show that  $r + tp$  is a primitive root of  $p^k$  for each  $k \geq 1$ .

4. Let  $n \in \mathbb{Z}^+$ . Prove that

- (a)  $p^n$  and  $2p^n$  have the same number of primitive roots.
- (b) Any primitive root  $r$  of  $p^n$  is also a primitive root of  $p$ .