
Math 4362 - Number Theory
Homework 6
Due in Class - Friday November 14, 2014

1. Find the order of all appropriate positive integers modulo 14.
2. Prove that
 - (a) if a has order hk modulo n , then a^h has order k modulo n .
 - (b) if a has order $2k$ modulo an odd prime p , then $a^k \equiv -1 \pmod{p}$.
3. Prove that
 - (a) the integer 2 has order n modulo $2^n - 1$.
 - (b) $\phi(2^n - 1)$ is a multiple of n for any $n > 1$.
4. If a has order h modulo n and if b has order k modulo n , when does ab have order hk modulo n ?
5. Let r be a primitive root of the integer n . Prove that r^k is a primitive root of n if and only if $\gcd(k, \phi(n)) = 1$.
6. Determine all primitive roots of $p = 19$.