
Math 4362 - Number Theory
Homework 5
Due in Class - Friday October 24, 2014

1. Using Fermat's little theorem, find the remainder on dividing

$$1865^{1910} + 1986^{2061}$$

by 7.

2. Use Wilson's Theorem to find the remainder when

(a) $15!$ is divided by 17.

(b) $2(26!)$ is divided by 29.

3. Calculate $\tau(5040)$, $\sigma(5040)$, $\mu(5040)$ and $\phi(5040)$.

4. Prove that if $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r} > 1$ then

$$\sum_{d|n} \mu(d)\sigma(d) = (-1)^r p_1 p_2 \cdots p_r.$$

5. Prove that for any positive integer n ,

$$\sum_{d|n} 1/d = \sigma(n)/n.$$

6. Find all solutions of $\phi(n) = 24$.

7. For positive integers m and n prove that

(a) $\phi(m)\phi(n) = \phi(mn)\phi(d)/d$, where $d = \gcd(m, n)$

(b) $\phi(m)\phi(n) = \phi(\gcd(m, n))\phi(\text{lcm}(m, n))$.