Math 4362 - Number Theory
Homework 3
Due in Class - Friday October 3, 2014

1. Determine all solutions in the integers of the following Diophantine equations

   (a) $24x + 138y = 18$

   (b) $14x + 35y = 93$

   (c) $54x + 21y = 906$

   (d) $158x - 57y = 7$

2. A man has $4.55 in change composed entirely of dimes and quarters. What are the maximum and minimum number of coins that he can have? Is it possible for the number of dimes to equal the number of quarters?

3. Prove each of the following for integers a, b and c:

   (a) If $a \equiv b \pmod{n}$ and $m \mid n$, then $a \equiv b \pmod{m}$.

   (b) If $a \equiv b \pmod{n}$ and $c > 0$, then $ca \equiv cb \pmod{cn}$

   (c) If $a \equiv b \pmod{n}$ and the integers a, b, n are all divisible by $d > 0$, then $\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{n}{d}}$

4. Use congruences to find the remainders when 2^{50} and 4^{65} are divided by 7?

5. Use congruences to show that 43 divides $6^{n+2} + 7^{2n+1}$ for any positive integer n.