Math 4362 - Number Theory
Homework 1
Due in Class - Friday 12 September 2014

1. Show that any integer of the form $6t+5$, for some integer t, is also of the form $3s+2$, for some integer s, but that the converse is false.

2. Use the Division Algorithm to establish that the fourth power of any integer is of the form $5k$ or $5k+1$, for some integer k.

3. Prove or disprove: if $a \mid (b+c)$ then $a \mid b$ or $a \mid c$.

4. Given integers, a, b, c, d verify that

(a) if $a \mid b$ then $a \mid bc$.

(b) if $a \mid b$ and $a \mid c$, then $a^2 \mid bc$.

(c) $a \mid b$ if and only if $ac \mid bc$, where $c \neq 0$.

(d) if $a \mid b$ and $c \mid d$, then $ac \mid bd$.

5. Use the Euclidean Algorithm to find $\gcd(a,b)$, and to obtain integers x and y such that $\gcd(a,b) = ax+by$, in the following cases:

(a) $a = 56$, $b = 72$.

(b) $a = 24$, $b = 138$.

(c) $a = 119$, $b = 272$.

(d) $a = 1769$, $b = 2378$.