Higher Math for Engineers Review for Exam III

The third exam will cover the topics below. It will consist of five problems including conceptual questions as usual. There will be five bonus problems spread over the test.

I. **Principle of superposition.** You must know how to solve non-homogeneous linear differential equations of second order using the principle of superposition.

Sample problems include: 4.5 # 1c), 2a), 6 (use the statement "Find the general solution to the given differential equation", particular solutions will not be given in the exam, you must find them as well)

II. Variation of parameters You must know how to solve non-homogeneous linear differential equations of second order using the method of variation of parameters.

Sample problems include: 4.6 #2,3,4,6,8

III. Laplace Transform. You must memorize the tables Linearity of the Laplace transform and Brief Table of Laplace Transforms and know how to use them in order to find the Laplace transform of a given expression.

Sample problems include: 7.2 # 14-20 (even)

IV. **Properties of the Laplace transforms.** You must know the table Properties of the Laplace Transforms and how to use it.

Sample problems include: 7.3 # 3,4,6

- V. Inverse Laplace Transform. Sample problems include: 7.4 # 1,2, 5,7
- VI. Solving initial value problems. You must know how to use the Laplace transform to solve an initial value problem. In the test, the partial fraction decomposition will be given but you should show all the steps that lead to the original fraction.

Sample problems include: 7.5 # 15,17,18,19.

- VII. **Conceptual problems.** You will be asked to answer some conceptual questions. The following are examples:
 - (i) According to the superposition principle, if y_1 is a solution to the differential equation

$$ay'' + by' + cy = f_1(t)$$

and y_2 is a solution to

$$ay'' + by' + cy = f_2(t)$$

and if c_1 , c_2 are arbitrary constants, what is a solution to

$$ay'' + by' + cy = c_1 f_1(t) + c_2 f_2(t)$$

(ii) In the method of variation of parameters, in order to solve the equation

$$ay'' + by' + cy = g(t)$$

one must first solve a linear system of two equations in the two variables v'_1 , v'_2 whose coefficients involve y_1 , y_2 , y'_1 and y'_2 .

- a) What do y_1 and y_2 represent?
- b) Write the linear system in v'_1 and v'_2 .
- (iii) Let $\mathcal{L}(f)(s)$ denote the Laplace transform of the function f, what is
 - a) $\mathcal{L}(f')(s)$?
 - b) $\mathcal{L}(f'')(s)$?
 - c) What is the Laplace transform of $e^{at}f(t)$?