Old Final

Use the reviews for exams I-III as well.

1. Use the Laplace transform to solve the intial value problem:

$$y'' + 4y' - 5y = te^t; \qquad y(0) = 1, \quad y'(0) = 0$$

You may use the partial fraction decomposition

$$\frac{s^3 + 2s^2 - 7s + 5}{(s+5)(s-1)^3} = \frac{35}{216}\left(\frac{1}{s+5}\right) + \frac{181}{216}\left(\frac{1}{s-1}\right) - \frac{1}{36}\left(\frac{1}{(s-1)^2}\right) + \frac{1}{12}\left(\frac{2}{(s-1)^3}\right)$$

but must show all the work needed to arrive to the fraction on the left.

2. A glass of tea at room temperature 70°F is chilled in ice (32°). If it takes 15 min for the tea to chill to 60°F, how long will it take for the tea to reach 56°F?

3. Find a *particular* solution to

$$y'' - y' + 9y = 3\sin(3t)$$

4. Classify the following differential equation as separable, linear or exact and solve it using the appropriate method.

$$(2x + y)dx + (x - 2y)dy = 0$$

5. Use the appropriate substitution to solve the equation:

$$\frac{dy}{dx} = \sqrt{x+y} + 1$$

6. Use the method of variation of parameters to find a *particular* solution of

$$y'' - 6y' + 9y = t^{-3}e^{3t}$$

Hint. Cancelling e^{3t} at some point will simplify the calculations. Use the convolution theorem to find the inverse Laplace transform $f(t)$ of

$$F(s) = \frac{14}{(s+2)(s-5)}$$

7. Solve the homogeneous equation:

$$(x^2 + y^2)\, dx + 2xy\, dy = 0$$

8. Use the method of superposition to find the *general* solution of:

$$y'' + 2y' + y = t^2 + 1 - e^t$$