Practice Problems for Exam II

The test will cover Sections 2.9-2.11, 3.1-3.6, 3.8 and 3.10.

On each proof clearly state the definitions involved.

When proving results which are already proved in the book, make sure to explain any details that may have been left out in the book.

The exam will consist of four of these problems:

1. Prove that if $\{S_n\}_{n=1}^{\infty}$ is bounded above and has a subsequence that is bounded below by A then $\limsup_{n\to\infty} S_n \geq A$.
2. Prove that if $\{S_n\}_{n=1}^{\infty}$ has no subsequence that is bounded below then $\limsup_{n\to\infty} S_n = -\infty$.
3. Prove that if $\{S_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers then $\liminf_{n\to\infty} S_n = \lim_{n\to\infty} S_n$.
4. Prove that if $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers and if $\liminf_{n\to\infty} S_n = \limsup_{n\to\infty} S_n = -\infty$, then $\{S_n\}_{n=1}^{\infty}$ diverges to $-\infty$.
5. Give a different proof of Theorem 2.10E using Cauchy sequences following this outline:
   (i) Show that, for any $N \in I$, the points $a_N, a_{N+1}, a_{N+2}, \ldots$ all lie in I_N.
   (ii) Use hypothesis (b) of the theorem to infer that $\{a_n\}_{n=1}^{\infty}$ is Cauchy and therefore convergent.
   (iii) Similarly, $\{b_n\}_{n=1}^{\infty}$ is convergent.
   (iv) From here, finish the proof as in 2.10E.
6. Show that no sequence that diverges to ∞ can be $(C, 1)$ summable.
7. Show that $\{\sin(n\theta\pi)\}_{n=1}^{\infty}$ is both divergent (this can be found in section 2.3) and $(C, 1)$–summable (this can be found in section 2.11). This gives an example of a $(C, 1)$–summable sequence which is not convergent in the usual sense.
8. Prove Theorem 2.11C.
9. Prove Theorem 3.1C.
10. Prove Theorem 3.2B.
11. Prove Corollary 3.3B.
12. Prove Theorem 3.6B.
13. Prove Theorem 3.10E.
14. Homework problems 2.10-2.11, 3.1-3.6, 3.8 and 3.10 without *.