Review for Exam II

1. Answer the following questions:

   a) What is a subspace of a vector space V? You must indicate the three properties involved.

   b) What is a basis of a vector space?

   c) What is the dimension of a vector space with a finite basis?

   d) In P_3, give an example of a spanning set which is not a basis. You must justify your answer, that is why the given vectors are a spanning set but not a basis.

   e) In $\mathbb{R}^{2\times 2}$ give an example of a linearly independent set which is not a basis. You must justify your answer, that is why the given vectors are linearly independent but not a basis.

   f) What is the Wronskian of a set of functions $\{f_1, \ldots, f_n\}$ in $C^n[a, b]$?

2. Pages 143-144 Problems: 9,10, 11, 13, 14, 15, 18.

3. Decide whether each of the following sets S is a subspace of the given vector space V. If it is a subspace, prove that it satisfies the three conditions. If not a subspace, state which of the conditions fails.

   a) $S = \left\{ \begin{pmatrix} x & x-2 \\ y & z \end{pmatrix} \text{ such that } x, y, z \in \mathbb{R} \right\}$, $V = \mathbb{R}^{2\times 2}$.

   b) $S = \{0, 1\}$, $V = \mathbb{R}$.

   c) $S = \{ax^2 + 2ax + 3a, \text{ such that } a \in \mathbb{R}\}$, $V = P_3$.

4. Pages 125-126: 5a),b),c), 6,8,9,14.

5. Find a basis for the nullspace of the following matrices:

   a) $\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & -1 & -1 & 2 \\ 3 & 2 & 1 & 1 & 3 \\ 3 & 6 & -1 & -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & -3 & 1 & 1 \\ 2 & 1 & -1 & 2 \\ 1 & 4 & -2 & 1 \\ 5 & -8 & 2 & 5 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 1 & -1 \\ 4 & -3 & 3 \end{pmatrix}$

6. Find a basis for the span of the following set of vectors: *Hint.* Put the vectors as rows in a matrix and reduce it to echelon form. The vectors in the original set that correspond to nonzero rows in the echelon form are a basis for the span.

   a) $\{(1, -1, 2, 5)^T, (0, -1, 3, 1)^T, (3, -4, 9, 16)^T, (1, 1, 0, 0)^T\}$

   b) $\{(1, 1)^T, (-1, 1)^T, (3, 2)^T\}$

   c) $\{(1, -1, 1)^T, (2, 0, 1)^T, (4, -2, 3)^T\}$

7. Decide whether the following vectors span the given vector space V:

   a) $\{(1, 2)^T, (2, 4)^T, (5, 10)^T\}$, $V = \mathbb{R}^2$

   b) $\{(1, 1)^T, (0, 1)^T, (-2, 2)^T\}$, $V = \mathbb{R}^2$

   c) $\{x, x + x^2, -x^2\}$, $V = P_3$

d) $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$, $V = \mathbb{R}^{2\times 2}$

8. Decide whether the following vectors are linearly dependent in the given vector space. If they are dependent find a nontrivial linear combination that gives the zero vector. If they are independent justify your answer.

   Note. A nontrivial linear combination is one in which the coefficients are not all zero. For example, $-2 \cdot (1,3) + \frac{1}{3}(6,18) = (0,0)$ is a nontrivial linear combination of (1,3) and (6,18) giving the zero vector.

   a) $\{(1,2,4)^T, (2,1,3)^T, (4,-1,1)^T\}$ in $\mathbb{R}^3$
   b) $\{x^2 - 2x + 3, 2x^2 + x + 8, x^2 + 8x + 7\}$ in P_3
   c) $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} \right\}$ in $\mathbb{R}^{2\times 2}$

9. Determine whether the following vectors form a basis for the given vector space. Justify your answer.

   Hint. Notice that if you are given n vectors in a vectors space V of dimension n, then they form a basis if and only if they are linearly independent. So in the cases below, you only need to check whether the vectors are linearly independent.

   a $\{1 - x, 1 + x, 1 - x^2\}$ in P_3
   b $\{(-1,1,0,0)^T, (0,-1,1,0)^T, (0,0,-1,1)^T, (-1,0,0,1)^T\}$ in $\mathbb{R}^4$
   c $\{(2,1,0)^T, (1,2,3)^T, (0,0,-1)^T\}$ in $\mathbb{R}^3$