Mathematics 2360 Name (please print) ______________________

Quiz 8-a

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I. Let A be a fixed matrix in $\mathbb{R}^{n\times n}$ and let S be the set of all matrices that commute with A; that is,
(2)

$$S = \{B \mid AB = BA\}.$$

Show that S is a subspace of $\mathbb{R}^{n\times n}$

$AI = IA = A$ where I is the $n\times n$ identity matrix,
so $I \in S$ and S is nonempty.

Let $B_1, B_2 \in S$
Then $(B_1 + B_2)A = B_1A + B_2A = AB_1 + AB_2 = A(B_1 + B_2)$
so $(B_1 + B_2) \in S$ and S is closed under addition.

Let $B \in S$ and $\alpha \in \mathbb{R}$
Then $(\alpha B)A = (\alpha I)(BA) = (\alpha I)(AB) = A(\alpha I)B = A(\alpha B)$
so $(\alpha B) \in S$ and S is closed under scalar multiplication.

$\therefore$ S is a subspace of $\mathbb{R}^{n\times n}$

II. (2) Determine the subspace of $\mathbb{R}^{2\times 2}$ consisiting of all matrices that commute with the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

$$S = \left\{ B \in \mathbb{R}^{2\times 2} \;\middle|\; \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} B = B \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}$$

Let $B = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x+z & y+t \\ z & t \end{pmatrix} = \begin{pmatrix} x & x+y \\ z & z+t \end{pmatrix}$$

Two matrices are equal iff their corresponding entries are equal.

$x+z=x$ $\quad$ $x+y=y+t$

$z=0$ $\quad$ $x=t$ $\quad$ y is free

$z=z$ $\quad$ $z+t=t$

So $\quad S = \left\{ \begin{pmatrix} x & y \\ 0 & x \end{pmatrix} \;\middle|\; x, y \in \mathbb{R} \right\}$