Mathematics 2360 Name (please print) ____________

Quiz 7-a

Mar 11, 2010

I. (3) Determine the nullspace of the matrix $A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$.

$N(A) = \{x : Ax = 0\}$

$\begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & 3 & 1 \\ -1 & -1 & 0 & -5 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Free variables: x_2, x_4

$x_2 = s \quad x_4 = t$

$x_1 = -x_2 - 5x_4 = -s - 5t$

$x_3 = -3x_4 = -3t$

$N(A) = \{(-s-5t, s, -3t, t) : s, t \in \mathbb{R}\}$

$= \text{span}\{(-1,1,0,0), (-5,0,-3,1)\}$

II. (2) For each of the following sets S, determine whether it is a subspace of the corresponding vector space V. Clearly justify your answers.

a) $S = \{p(x) \in P_3 \,|\, p(0) = 0 \text{ or } p(1) = 0\}$, $V = P_3$.

b) $S = \{f \in C[-\pi, \pi] \,|\, f(-x) = f(x)\}$, $V = C[-\pi, \pi]$. That is, S consists of the even functions in $C[-\pi, \pi]$.

a) $p_1(x) = x - 1$ satisfies $p_1(1) = 0$ so S is nonempty.
$p_2(x) = -x$ satisfies $p_2(0) = 0$, but $(p_1 + p_2)(0) = p_1(0) + p_2(0) = -1 + 0 = -1 \neq 0$
and $(p_1 + p_2)(1) = p_1(1) + p_2(1) = 0 + -1 = -1 \neq 0$

So S is not closed under addition and S is not a subspace of V

b) $f(x) = 0$ satisfies $f(x) = f(-x)$, so S is nonempty.
Let $g, h \in S$ and $\alpha \in \mathbb{R}$
$(\alpha g)(x) = \alpha[g(x)] = \alpha[g(-x)] = (\alpha g)(-x)$ so $\alpha g \in S$
$(g+h)(x) = g(x) + h(x) = g(-x) + h(-x) = (g+h)(-x)$ so $g + h \in S$

So S is a subspace of V