Mathematics 2360 Name (please print) ____________________

Quiz 7-a

Mar 11, 2010

I. (3) Determine the nullspace of the matrix $A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$.

$N(A) = \{x : Ax = 0\}$

$\begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & 3 & 1 \\ -1 & -1 & 0 & -5 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Free variables: x_2, x_4

$x_2 = s \quad x_4 = t$

$x_1 = -x_2 - 5x_4 = -s - 5t$

$x_3 = -3x_4 = -3t$

$N(A) = \{(-s-5t, s, -3t, t) : s, t \in \mathbb{R}\}$

$= \text{span}\{(-1,1,0,0), (-5,0,-3,1)\}$

II. (2) For each of the following sets S, determine whether it is a subspace of the corresponding vector space V. Clearly justify your answers.

a) $S = \{p(x) \in P_3 \mid p(0) = 0 \text{ and } p(1) = 0\}$, $V = P_3$.

b) $S = \{f \in C[0,1] \mid f(y) \leq f(x) \text{ for } x \leq y\}$, that is, S is the set of continuous nonincreasing functions on $[0,1]$, $V = C[0,1]$.

a) $p(x) = 0 \in S$ so S is nonempty

Let $p_1, p_2 \in S$ and $\alpha \in \mathbb{R}$

$p_1(0) + p_2(0) = 0 + 0 = 0$ so $(p_1 + p_2)(0) = 0$

$p_1(1) + p_1(1) = 0 + 0 = 0$ so $(p_1 + p_2)(1) = 0$ and $p_1 + p_2 \in S$

$(\alpha p_1)(0) = \alpha[p_1(0)] = \alpha \cdot 0 = 0$

$(\alpha p_2)(1) = \alpha[p_1(1)] = \alpha \cdot 0 = 0$ so $\alpha p_1 \in S$

$\therefore S$ is a subspace of V

b) $f(x) = 0$ satisfies $f(x) \geq f(y)$ for $x \leq y$ so S is nonempty.

If $f(x) \geq f(y)$ for $x \leq y$ and $\alpha < 0$

$\alpha f(x) \leq \alpha f(y)$ for $x \leq y$

$\therefore S$ is not closed under scalar multiplication and S is not a subspace of V