Mathematics 2360 Name (please print) ____________________

Quiz 11

May 4, 2010

I. (3) Find a matrix A such that the linear operator L on $\mathbb{R}^3$ given by $L((x_1, x_2, x_3)^T) = (x_3 - 4x_2 + x_1, x_1, x_2 - x_1)^T$ satixfies $L(\mathbf{x}) = A\mathbf{x}$.

$$a_1 = L(e_1) = L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$a_2 = L(e_2) = L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$$a_3 = L(e_3) = L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

II. (2) Let S be the subspace of $\mathbb{R}^3$ spanned by $\mathbf{x} = (1, 2, -4)^T$. Find a basis for $S^\perp$.

$$S^\perp = \{ y \mid x^T y = 0 \}$$

$$\begin{bmatrix} 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

y_2 and y_3 are free

$y_1 = -2s + 4t$

$y_2 = s$

$y_3 = t$

So a basis for $S^\perp$ is

$\{(-2,1,0)^T, (4,0,1)^T\}$

III. (3) Find all the least square soloutions of the system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 0 \\ 8 \end{pmatrix}$.

$$A^TA = \begin{bmatrix} 6 & 12 \\ 12 & 24 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

$$A^TAx = A^Tb \Rightarrow \begin{bmatrix} 6 & 12 \\ 12 & 24 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 12 & -10 \\ 12 & 24 & -20 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 6 & 12 & -10 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 3 & 6 & -5 \\ 0 & 0 & 0 \end{array}\right]$$

x_2 is free

$x_1 = -\frac{5}{3} - 2t$

$x_2 = t$

$\therefore$ The least square solution is $\{(-\frac{5}{3}-2s, s)^T \mid s \in \mathbb{R}\}$