Mathematics 2360 Name (please print) ____________

Quiz 10-a

Apr 22, 2010

I. (4) Find a basis for the row space, a basis for the colum space and a basis for the nullspace of $\begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}$

$$\begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -\frac{10}{7} \\ 0 & 1 & 0 & -\frac{2}{7} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

So a basis for the

- row space is $\{(-3,1,3,4), (1,2,-1,-2), (-3,8,4,2)\}$
- column space is $\{(-3,1,-3)^T, (1,2,8)^T, (3,-1,4)^T\}$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{10}{7} & 0 \\ 0 & 1 & 0 & -\frac{2}{7} & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \Longrightarrow \begin{array}{l} x_1 = \frac{10}{7}t \\ x_2 = \frac{2}{7}t \\ x_3 = 0 \\ x_4 = t \end{array}$$

and $\{(\frac{10}{7}, \frac{2}{7}, 0, 1)^T\}$ is a basis for the nullspace

II. (2) Determine whether the vector $\mathbf{b} = (2,5,2)^T$ is in the column space of $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$. Justify you answer.

$$b = 5 \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix}$$

So b is in the column space of A.

Alternatively:

$$A\vec{x} = \vec{b} \Rightarrow \left(\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 0 & 5 \\ 0 & 1 & 2 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$ is consistent

so b is in the column space of A.