Colloquium. Department of Mathematics and Statistics.
Texas Tech University.

Non-linear Problems in Fluid Dynamics

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ABSTRACT. This talk discusses several problems and results on non-linear flows in fluid dynamics. The first part is devoted to clear fluids and is focused on three-dimensional (3-D) Navier-Stokes equations (NSE). The second part is devoted to Forchheimer flows in porous media for both single and multi phase problems.

Part IA. We consider the incompressible NSE with potential body forces in the 3-D torus. For the long-time dynamics, we study the normalization introduced by Foias-Saut. We construct a suitable Banach space such that the normalization map is continuous, and the normal form is a well-posed system. Furthermore, we show that this normalization produces a Poincaré-Dulac normal form which is obtained by an explicit change of variable. A family of homogeneous gauges is used to estimate involved homogeneous polynomials in infinite dimensional spaces.

Part IB. We study incompressible fluids in a thin domain which arise from oceanography and meteorology. The fluid is subject to the Navier friction boundary condition. For the two-layer fluids, the interface boundary condition from the coupled atmosphere and ocean model is imposed. We show that regular solutions exist for all time when the initial data and body force belong to large sets in relevant function spaces. The estimates obtained reflect the connection between the non-linear terms in NSE with the boundary conditions and the boundary's non-trivial geometry.

Part IIA. We generalize Darcy's law and common Forchheimer's laws to model non-linear flows in porous media. For slightly compressible fluids, the problem is reduced to a degenerate parabolic equation for the pressure. The solutions are estimated, particularly for large time, in supremum norm and different Sobolev norms. They are showed to be continuously dependent on the initial and boundary data, and the Forchheimer polynomials. Various techniques from partial differential equations and dynamical systems are used and combined with the special structure of the equation in our analysis.

Part IIB. We model multi-dimensional two-phase flows of incompressible fluids in porous media using generalized Forchheimer equations and the capillary pressure. We find a family of steady state solutions with certain geometric properties. To study their stability, the linearized system is derived and reduced to a parabolic equation for the saturation. This equation has a special structure depending on the steady states which we exploit to prove the stability results as well as the solutions' qualitative properties.