Texas Tech University. Applied Mathematics Seminar.

FROM FUNCTIONAL ANALYSIS TO ITERATIVE METHODS

Robert Kirby, Texas tech University

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ABSTRACT. We examine condition numbers, preconditioners, and iterative methods for finite element discretizations of coercive PDE in the context of the fundamental solvability result, the Lax-Milgram Lemma. Working in this Hilbert space context is justified because finite element operators are restrictions of Hilbert space operators to finite-dimensional subspaces. Moreover, useful insight is gained as to the relationship between Hilbert space and matrix condition numbers, and translating Hilbert space fixed point iterations into matrix computations provides new ways of motivating and explaining some classic iteration schemes. In this framework, the "simplest" preconditioner for an operator from a Hilbert space into its dual is the Riesz isomorphism. Simple analysis gives spectral bounds and iteration counts bounded independent of the finite element subspaces chosen. Moreover, the abstraction allows us not only to consider Riesz map preconditioning for convection-diffusion-reaction equations in H^1, but also operators on other Hilbert spaces, such as planar elasticity in $\left(H^1\right)^2$.