Texas Tech University. Applied Mathematics Seminar.

FINITE ELEMENT MATRICES AND BERNSTEIN POLYNOMIALS

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Room: MA 111, Time: 4:00pm

ABSTRACT. High-degree finite element methods are typically only made efficient in terms of computational cost and memory usage by finding special structure that allows operators to be applied with fast algorithms. In rectangular element domains, one can frequently build basis functions by tensor products of one-dimensional polynomials. This can lead to efficient implementations.

On simplicial element domains, however, far fewer fast algorithms are known. Special bases may be carefully constructed, but these often lack certain desireable properties (such as rotational symmetry). Working toward the goal of fast, general-purpose simplicial bases with good symmetry, I will work the Bernstein polynomials.

For polynomial degree n, I will show that finite element mass and stiffness matrices in 1d may be applied in $O(n \log(n))$ operations with $O(n)$ local store if Bernstein bases are used. Moreover, I will suggest how on the simplex in dimension $d \geq 2$, it is possible to apply these operators in $O(n^{1+1/d} \log(n))$, which up to the log factor is the same complexity as rectangular domains with tensor products. The trick is Hankel matrices, which up to row ordering admit a circulant embedding and hence fast application via the FFT.

This work is preliminary and still theoretical. I believe there are very good opportunities for student projects here, both in terms of implementation and extending the results to other kinds of finite element bases and operators.