Partial Fractions

These notes are concerned with decomposing rational functions

$$\frac{P(s)}{Q(s)} = \frac{a_M s^M + a_{M-1}s^{M-1} + \cdots + a_1 s + a_0}{s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0}$$

Note: we can (without loss of generality assume that the coefficient of s^N in the denominator is 1.

I. **Degree $\boldsymbol{P}(s)$ > Degree $\boldsymbol{Q}(s)$** In this case first carry out long division to obtain

$$\frac{P(s)}{Q(s)} = P_1(s) + \frac{P_2(s)}{Q(s)}$$

where Degree(P_2) < Degree(Q).

II. **Nonrepeated factors**

If $Q(s) = (s - r_1)(s - r_2)\cdots(s - r_n)$ and $r_i \neq r_j$ for $i \neq j$

$$\frac{P(s)}{Q(s)} = \frac{A_1}{(s - r_1)} + \frac{A_2}{(s - r_2)} + \cdots + \frac{A_n}{(s - r_n)}$$

III. **Repeated Linear Factors**

If $Q(s)$ contains a factor of the form $(s - r)^m$ then you must have the following terms

$$\frac{A_1}{(s - r)} + \frac{A_2}{(s - r)^2} + \cdots + \frac{A_m}{(s - r)^m}$$

IV. **A Nonrepeated Quadratic Factor**

If $Q(s)$ contains a factor of the form $(s^2 - 2\alpha s + \alpha^2 + \beta^2) = (s - \alpha)^2 + \beta^2$ then you must have the following term

$$\frac{A_1 s + B_1}{(s^2 - 2\alpha s + \alpha^2 + \beta^2)}$$

V. **Repeated Quadratic Factors**

If $Q(s)$ contains a factor of the form $(s^2 - 2\alpha s + \alpha^2 + \beta^2)^m$ then you must have the following terms

$$\frac{A_1 s + B_1}{(s^2 - 2\alpha s + \alpha^2 + \beta^2)} + \frac{A_2 s + B_2}{(s^2 - 2\alpha s + \alpha^2 + \beta^2)^2} + \cdots + \frac{A_m s + B_m}{(s^2 - 2\alpha s + \alpha^2 + \beta^2)^m}$$